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Groundwater Pollution Diffusion Model Based on Partial Differential Equation

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ABSTRACT

The objective of the paper is to solve the problems of groundwater using partial differential equation (PDE). The finite element method is one of the most important solutions to the problems, which is applied to obtain the approximate solution of functions. The paper has applied the toolbox method to solve the problems of groundwater in the engineering of the planar two-dimensional (2D) steady flow and the planar 2D unsteady flow. In addition, the planar 2D steady flow includes the specific problems of fully penetrating well with the preset depth of the confined aquifer and the steady flow of the unconfined aquifer. Besides, the PDE toolbox has been applied to solve the practical groundwater problems in engineering, the results have shown that in terms of solving the groundwater problems, the MATLAB PDE toolbox is more convenient, simple, and accurate compared with the method of directly programming the original program. Therefore, in case of problems that cannot be solved by the graphical user interface of the PDE toolbox, the command functions in the MATLAB toolbox could be applied to perform numerical calculations on the problems.

INTRODUCTION

In recent years, the significant development of the science and technology and the agriculture and industry, the continual expansion of cities, the rapid growth of population, and the massive pollution and waste of water resources have caused the continuous decrease of groundwater level, leading to the ever-reducing water resources for human consumption; consequently, water supply crises at different levels have appeared all around the world (Augeraud-Véron et al. 2017). Besides, as more mines are mined, the groundwater resources are also reduced. With the occurrence of the underground sewage incident brewed by a company in Weifang City, Shandong Province, China, people are more concerned about groundwater pollution (Wang & Gao 2017). According to national surveys, given the serious groundwater pollution, the situation of groundwater pollution has become very serious, and the speed of water pollution has been increasing. Being an important natural water resource, groundwater is closely related to the daily lives of human beings; it is the only water supply in some cases (Meng et al. 2017, Nnolim 2017). Therefore, the rational development and utilization of groundwater resource have become a problem that must be solved at present.

Currently, the mathematical models of more and more engineering and scientific problems could be classified as the determining solution of partial differential equation (PDE) (Bo et al. 2018). The processes of solving the determining solutions of PDE are relatively complicated; the research on numerical methods of PDE has become the primary researching directions of PDE, as well as the core content of engineering and scientific calculations.

The partial differential equation toolbox (PDE toolbox) software has provided the researchers with a practical working environment and simple solution methods. The toolbox software is specially designed for both beginners and senior users (Wei et al. 2017). There are two applications of the toolbox software on PDE-associated problems: one is the direct application of the graphical user interface (GUI) in the PDE toolbox software. Enter "pdctool" on the command line of the MATLAB window and run it (Abdelrahman et al. 2017). The graphical user interface of the PDE toolbox would be automatically generated, which is a separate graphical environment for solving problems with partial differential equations (Martelloni et al. 2017). Common applications could use specific physical conditions rather than abstract coefficients. The utilization of the pdctool requires no mathematical knowledge of MATLAB (Kovacic et al. 2017) in terms of solving the partial differential equations, it could be performed under the guidance of examples. The graphical user interface window allows following operations: drawing the geometric area of the partial differential equation, setting the type of the equation and the parameters of the equation, setting the type and parameters of the boundary condition, meshing the geometric area, solving the equation, and plotting the chart.

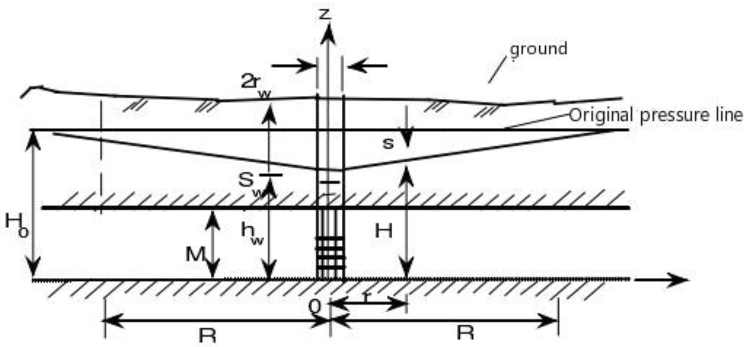

Fig. 1: Complete aquifer of the confined aquifer.

MATERIALS AND METHODS

For advanced non-standard applications, it is possible to describe areas and boundary conditions in the MATLAB workspace. The toolbox function of partial differential equations is applied to manage data on unstructured grids, generate meshes, and perform finite element method discretization of partial differential equations, etc. While applying the graphical user interface of PDE toolbox software to solve the numerical solutions, certain difficulties could appear, therefore, a solver could be designed to solve it, or the finite element method (FEM) could be applied to solve the non-standard problem of more complex algorithms.

As Fig. 1 shows, the fully penetrating well is of the confined aquifer. The well is homogeneous and isotropic, its thickness is M, its osmotic coefficient is K, the original water level of the confined aquifer is H_0, after the pumping reaches a stable state, the water level in the well is h_w, the radius of the water filter pipe is r_w, and the influence radius of pumping well is R hypothetically. The values of coefficients mentioned above are as follows: W = 0.8m/d; M = 100m; R = 300m; H_0 = 170m; r_w = 80m; h_w = 120m.

One of the basic types of partial differential equations solved by the PDE toolbox is the elliptic equation, whose basic form is:

$$-\nabla \cdot (c\nabla u) + au = f, in \ \ \Omega \qquad \ldots(1)$$

Where, Ω is the planar area. The equation coefficient c, a, and f as well as the unknown function u are all real functions (or complex functions) defined on the Ω.

The boundary conditions of PDE are:

1. Dirichlct boundary condition

   $hu = r$

2. Ncumann boundary condition

   $\vec{n} \cdot (c\nabla u) + qu = g$

3. Hybrid boundary condition: the combination of Dirichlct boundary condition and Ncumann boundary condition.

Where the $\vec{n}$ is the normal vector out of the unit on the $\partial\Omega$, the boundary condition coefficient h, r, q, and g are the functions on the $\partial\Omega$. Note: In the finite element method, the Dirichlet boundary condition could also be called as the essential boundary condition (or the first type of boundary condition), and the Neumann boundary condition could also be called as the natural boundary (or the second type of boundary condition).

The equation for the groundwater problem in engineering is:

$$\frac{\partial}{\partial x}\left(KM\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(KM\frac{\partial H}{\partial y}\right) = 0 \qquad \ldots(2)$$

The Equation (2) of the problem is an elliptic partial differential equation, whose corresponding coefficients c, a, and f in the elliptic basic equations are c = KM, a = 0, and f = 0, respectively. The area Ω is the bounded area of the ring-like plane; in addition, the boundaries of the two circles in the ring are the first type of boundary conditions (the Dirichlet boundary conditions). In the area of $x^2 + y^2 = R^2$, the coefficients h and r are respectively h=1 and r=170; in the area of $x^2 + y^2 = r_w^2$, the coefficients h and r are respectively h = 1 and r = 120.

RESULTS AND DISCUSSION

Planar two-dimensional steady flow problem: Fig. 2 is a planar graph where two rivers meet and the two faults intersect to form a trench-shaped valley. The South fault is a descending fault, which tends to the Northeast, and its South side is a giant granite wall; the West fault is also a descending fault, which tends to the Southeast, and its West side is also a giant granite wall. On the East and North sides of the area,

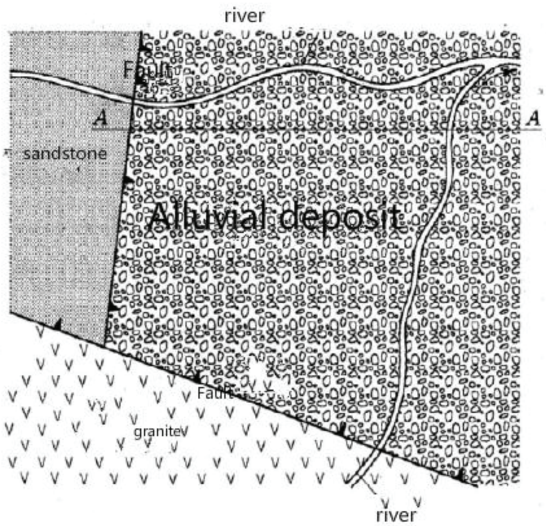

Fig. 2: Floor plan of the valley area.

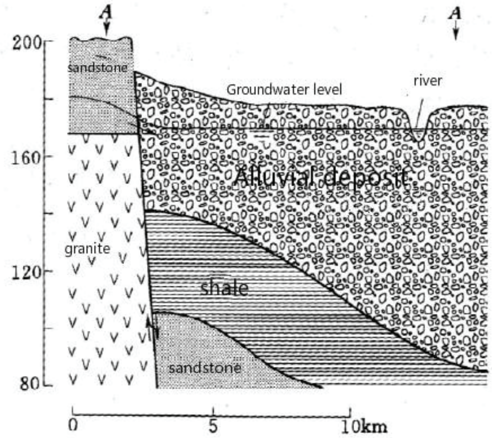

Fig. 3: Geological section of the valley area.

two respective tributaries of the East River and the North River pass through, merging into the main river channel in the Northeast corner. Fig. 3 is the geological section of the valley area. It can be inferred from Fig. 3 that the west side of the valley is a granite wall, whose top is covered with a layer of aquiferous sandstone, with a small amount of water supplying to both the alluvium and the sedimentary layers in the long-term; in the river valley, the bottom of the alluvium and the sedimentary layers are dense layers of shale; in the middle and lower reaches of the river valley, the top of the alluvium and sedimentary aquifers is covered with an impervious overburden layer, whose thickness is about 8 m.

Fig. 4 is the longitudinal section of the aquifer. It is assumed that the shape of the section that is parallel to the direction of the West fault line is constant, therefore, the variation of the alluvium and the sedimentary aquifers in the entire area could be observed. Fig. 5 is the vertical slope line of the two rivers. The annual average water level at the intersection of the two rivers is 168.0m; starting from the average level and pushing back to the upstream of the two rivers at the same time, in accordance with the actual measurement data of the water levels of the river control points, the surface longitudinal slopes of the rivers are consistent.

The upper reach of the river valley area, i.e. the East side of the West fault, is the irrigation channel area of the river. In accordance with the descriptions above, the boundary conditions and supplying conditions are: the East River and the North River are two known head boundaries; the South fault is granite, therefore, it could be regarded as an impervious boundary; the supply boundary of the groundwater in the Western sandstone layer is the boundary of the known supply flow; the shale layer at the bottom of the alluvium and the sedimentary aquifer is the impervious bottom layer; the upper reach of the valley is the precipitation supply rate ε and the area receiving irrigation. Due to the impervious cover layer, the remained water of irrigation and precipitation is the surface runoff that doesn't participate in the flow of groundwater in the aquifer.

It can be inferred from the descriptions of the boundary conditions that the South fault is an impervious boundary, thus, the flow of boundary inflow supply QC is 0; the West fault is the known supply flow boundary, the flow of boundary inflow supply QC, based on the actual measurement and estimation, is 10L/(s·km) on average annually, and $QC=0.864m^3/(d\cdot m)$ could be obtained through unit conversion.

Through the description of groundwater problems in engineering, it can be inferred that the groundwater problem should be a planar two-dimensional (2D) steady flow osmotic problem. In accordance with the derivation of the basic equation of groundwater, it is observed that in terms of the steady flow problems, the equations of both unconfined flow and confined flow could be written in the following basic form:

$$\frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial X}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial H}{\partial y}\right)+\varepsilon=0 \qquad \ldots(3)$$

If the steady flow is the confined flow, in Equation (3), T = kh, where, k is the osmotic coefficient, k = 50; besides, h is the depth of water, h = 80, T = 4000. If the steady flow is the unconfined flow, in Equation (3), T = kM, where k is the osmotic coefficient and M is the thickness of the aquifer.

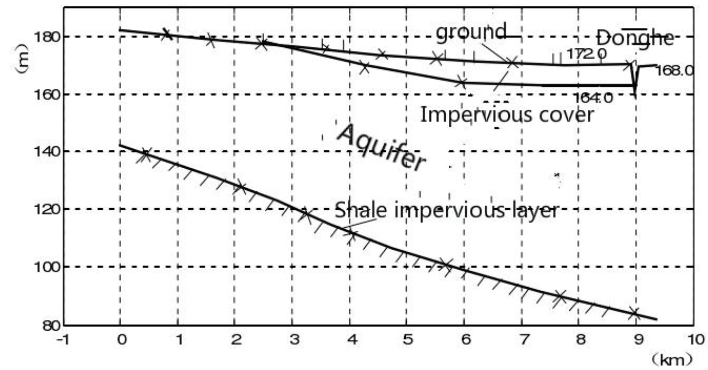

Fig. 4: The longitudinal section of the aquifer.

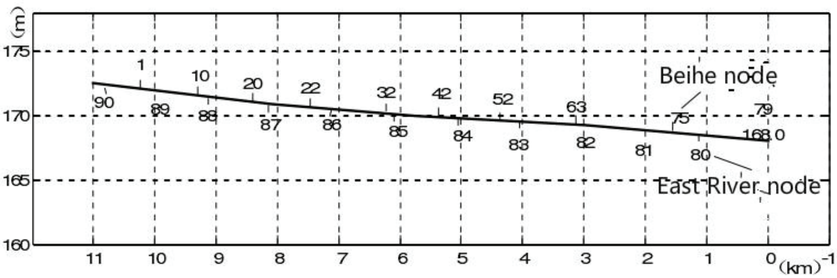

Fig. 5: Two rivers vertical slope line.

The parameter ε in the equation is the stable vertical osmotic supply rate occurring in the irrigation section, ε = 0.0003 m/d for the problem. For the impervious layer covered sections without irrigation and precipitation, the basic equation could be expressed as:

$$\frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial X}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial H}{\partial y}\right)=0 \qquad \ldots(4)$$

Planar two-dimensional unsteady flow problem: The planar two-dimensional unsteady flow problem has been deduced in the books of engineering groundwater. In terms of the unsteady flow problem, whether it is the unconfined flow or the confined flow, the basic equations could be expressed as:

$$\frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial H}{\partial y}\right)+\varepsilon=S\frac{\partial H}{\partial t} \qquad \ldots(5)$$

If the unsteady flow is the unconfined flow, in the Equation (5), T = ch, S = μ, where K is the osmotic coefficient, k = 50; h is the depth of water, h = 80, therefore, T = kh = 4000, μ is the yield of water, μ = 0.17. If the unsteady flow is the confined flow, in the equation, T = kM, S = $S_S \cdot M$, where, K is the osmotic coefficient, S_S is the vertical hydraulic conductivity that approximates 0 here, M is the thickness of the aquifer.

For the sections that are covered with impervious layers (or the non-irrigation and non-precipitation sections), the basic equation is:

$$\frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial H}{\partial y}\right)=S\frac{\partial H}{\partial t} \qquad \ldots(6)$$

The equation for such groundwater problems in engineering belongs to the parabolic partial differential equations. The basic form of the parabolic equation is:

$$d\frac{\partial u}{\partial t}-\nabla\cdot\left(c\nabla u\right)+au=f \qquad \ldots(7)$$

The c, a, f, and d in the equation corresponds to equation (6), c = T = 4000, a = 0, f = 0, and d = S = μ = 0.17.

Plane three-dimensional groundwater problem: The groundwater aquiferous system in nature is often a transboundary aquifer system that contains various aquitards and aquifers. While pumping water from a certain aquifer, variations would happen to the aquifer and its adjacent aquitards and aquifers. Since the whole system is real and the groundwater flow field is three-dimensional, theoretically, the closest situation to the actual situation is to establish a three-dimensional mathematical model for the multi-layer aquifer system, the aquitards and the aquifers in the three-dimensional groundwater aquiferous system with the porous medium are generally in alternative layer distribution. The clay with a small osmotic coefficient and the sand with a large osmotic coefficient respectively constitute the aquitard and the aquifer, therefore, the parameters in the vertical direction are mutagenic. Meanwhile, each aquitard and each aquifer simultaneously on the plane is often heterogeneous, hence, the parameters are gradual in the horizontal direction. The PDE toolbox method could be applied to solve three-dimensional groundwater flow problems.

M file for PDE: In terms of the certain practical problems of engineering and scientific fields, it is often necessary to communicate with other finite elements and plotting software to complement the solutions.

The most commonly applied method is to convert each other through DXF files, while the transmission requires graphics. The DXF file is a graphical interactive file. It is a graphical interactive file that can be accepted by software such as MICROSTATION, SSAP, and AUTOCAD. DXF is a writable ASCII format. The M file and DXF file format in PDE are identical. Therefore, if the geometric descriptions of the M file of the partial differential equation is generated and then written into the DXF physical segment, the interface of the M file of the partial differential equation to the DXF file could be formed; through reversing the process, the interface of the DXF file to the M file of the partial differential equation could be formed. In terms of solving the problems by using the graphical user interface of the toolbox software, the M file of the solving processes could be generated by clicking the command "Save As" under the menu File to open the Save As dialogue box, selecting the save path of the file, naming the file such as pmwdl input by the keyboard, and then clicking the "Save" button to generate the file pmwdl.m. M files of such type could be saved and opened through the file menu. The MATLAB function is not an M file of the script type. Such kind of file avoids the use of functions and name conflicts between variables in the main workspace. The name of the file must match the model name so that it can be called by other programs.

CONCLUSION

The paper has mainly introduced the relevant knowledge principles of PDE toolbox and finite element method, as well as applying the graphical user interface (GUI) of PDE toolbox to groundwater. The paper has performed numerical calculations of problems including the planar two-dimensional steady flow and the planar two-dimensional unsteady flow of groundwater. It can be inferred from the problem-solving processes and results that the method proposed in the paper is simple and convenient in the application; besides, it also shows great advantages in both calculation accuracy and calculation efficiency. However, the method is also of several deficiencies including the difficulty in storing the solutions, which would in further affect the subsequent calculations. In addition, in the calculation processes of the inverse problems, it is necessary to continuously call the original files and apply the calculated results.

In terms of solving the problem of partial differential equations related to groundwater pollution, in addition to the graphical user interface of MATLAB PDE toolbox software, the command function in the toolbox software could also be applied to solve the problem by creating an M file that describes the geometry. There are some difficulties in solving specific problems which cannot be solved by using the PDE tool graphical user interface. For example, when the geometric areas are not composed of lines, arcs, elliptical arcs, and the combined graphics of them, in order to solve the problem, only the command function in the toolbox could be applied to perform the numerical calculations, which is much simpler and faster than programming the original program directly.

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