



## The Reverse Based Identification of Source Intensity Changes in Sudden Pollution Accidents in Medium River

Rui Huang\*(\*\*\*)†, Long-xi Han\*, Wen-long JIN\*, Hui Peng\*\*, Man-man PAN\*\* and Hong Zhang\*\*\*\*

\*College of Environmental Science and Engineering, Hohai University, Nanjing 210098, China

\*\*Bureau of Hydrology, Ministry of Water Resources, Beijing 100053, China

\*\*\*Yangzhou Vocational University, Yangzhou 225000, China

\*\*\*\*Bureau of Hydrology, Liaoning Fushun city, Fushun 113000, China

†Corresponding author: Rui Huang

Nat. Env. & Poll. Tech.  
Website: [www.neptjournal.com](http://www.neptjournal.com)  
Received: 25-10-2014  
Accepted: 23-11-2014  
**Key Words:**  
DE algorithm  
Inverse accuracy  
Inverse problem  
Water pollution accident  
Wide and shallow rivers

### ABSTRACT

Based on the migration and transformation features of pollution sources in wide and shallow rivers, several monitoring points of water quality were set in the downstream to obtain a series of monitoring information. The changes of source intensity were, therefore, determined to present the problems in the inversion of instantaneous pollution sources during sudden pollution accidents. The research employs a differential evolution (DE) algorithm to inverse the location and number of emissions of pollutants discharged. The results show that the DE algorithm can accurately identify the sizes and locations of pollution sources and provide technical support for the identification of pollution sources. The research explored the impact of monitoring schemes on the accuracy of inversion results. It is of technical significance for manipulating and optimizing the emergency monitoring plans.

### INTRODUCTION

In the research of mixed transport law of pollutants in water, the problem is mainly to obtain pollutants' spatial distribution and variation with time, when the control equation of environmental system (model structure and parameter), boundary conditions, and initial conditions are known. Inversely, the control equation of water environmental system (model structure and parameter), boundary conditions, initial conditions, etc. can be acquired according to the partial spatial and temporal distribution information of the pollutants (Jin & Zhou 1997 and Liu et al. 2009). The source intensity and location of pollutants are most difficult to be monitored when a sudden water pollution accident occurs (Araujo et al. 2014). This poses a great challenge to the water quality forecasting. How to inverse the complete process of pollution source emissions, after the accidents, based on the monitoring information in downstream is of great significance (Plummer & Long 2009). Jin & Chen (1992) studied the problems in the inversion of the source control using a pulse spectrum-optimization method. Han (2001) and Han et al. (2001) inverted the sources in areas of single rivers and water net areas. Min et al. (2008, 2009) investigated the problems in the inversion of identifying sources of one-dimensional

convection-diffusion equation using genetic algorithm and explored the parameter identification of one-dimensional convection-diffusion equation for multiple pollution sources, applying regularization and iterative method, using the regularization idea in inverse problems.

The research employed a DE algorithm as resolving method. Considering the impacts of hydrodynamic forces of the accident water area and the accurately quantitative description of water quality at monitoring points on the inverse accuracy, the research analysed the influences of monitoring schemes (i.e. the numbers of monitoring points and monitoring frequency) on the inverse accuracy.

### METHODS

#### Two-dimensional migration-diffusion model of instantaneous pollution sources in wide and shallow rivers:

To simplify the performance, the river is supposed to be  $L$  in length and the background concentration of the pollutants is 0. Assuming that, the pollutants with the mass of  $M$  were instantly discharged into the river at the coordinate of  $(x_m, y_m)$  when  $t = 0$ , the concentration increment of the pollutants varying with time and space can be described using the following convection-diffusion equation:

$$\left\{ \begin{array}{l} \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} (D_x \frac{\partial C}{\partial x}) + \frac{\partial}{\partial y} (D_y \frac{\partial C}{\partial y}) - \frac{\partial(u_x C)}{\partial x} - \frac{\partial(u_y C)}{\partial y} - KC + M\delta(x - x_m, y - y_m, t) \\ \text{Initial condition: } C(x, y, t)|_{t=0} = 0 \\ \text{Boundary condition: } C(x, y, t)|_{x=0} = C_0 \\ \frac{\partial C(x, y, t)}{\partial x} \Big|_{x=L} = 0 \end{array} \right. \quad \dots(1)$$

Where,  $C$  is the concentration increment of the pollutants;  $D_x$  and  $D_y$  are axial and lateral diffusion coefficients respectively;  $u_x$  and  $u_y$  are axial and lateral flow speeds respectively;  $K$  is the degradation coefficient of the pollutants;  $M\delta(x - x_m, y - y_m, t)$  is source term, where  $M$  is the intensity of instantaneous point sources,  $x_m$  and  $y_m$  are the axial and lateral coordinates of point sources in two-dimensional space, and  $\delta(\cdot)$  is Dirac Delta function.

Based on the known parameters and source items in equation 1, the definite solution constituted is forward solution, but the analytical solution is:

$$C(x, y, t) = \frac{M}{4\pi ht\sqrt{D_x D_y}} \exp\left[-\frac{(x - x_m - u_x t)^2}{4D_x t} - \frac{(y - y_m - u_y t)^2}{4D_y t}\right] \exp(-Kt) \quad \dots(2)$$

**Problems in the inversion of the two-dimensional source terms of the instantaneous pollution sources in wide and shallow rivers:** The intensity and location of pollution sources can hardly be monitored timely, when sudden water pollution accidents occur. As a result, it is difficult to obtain source term  $M\delta(x - x_m, y - y_m, t)$  in the control equation. To determine the pollutant source intensity  $M$  in the source term and the location and of the pollution sources, a known additional condition is given, that is, the monitoring information of pollutants concentration at several points in the downstream in a time series. It is described as:

$$C(x, y, t) = C_{obs}(x_i, y_i, t_{j_i}) \quad (i = 1, 2, \dots, n; j_i = 1, 2, \dots, k_i) \quad \dots(3)$$

Where  $C_{obs}(x_i, y_i, t_{j_i})$  is the monitored pollutants concentration at  $i^{th}$  monitoring point and  $j^{th}$  time series;  $k$  is the total number of monitoring points; and  $n_i$  is the monitoring frequency at  $i^{th}$  monitoring point.

To determine the source intensity  $M$  and location  $(x_m, y_m)$  of the pollution sources, according to the monitoring information of pollutants concentration, is in the inversion of the identification of source items. That is, to inverse the source intensity  $M$  and location  $(x_m, y_m)$  of pollution sources

in the control equation using information of the pollutants concentration monitored at several monitoring points in a time series of downstream. Suppose that  $S$  is the undetermined vector function constructed using source intensity  $M$  and location  $(x_m, y_m)$  and let the solution of  $S$  in the forward problem (1) be  $C(S, x, y, t)$ , the above inverse problem is transformed to the following optimization problem:

$$\min \sum_{i=1}^n \sum_{j_i=1}^{k_i} [C(x_i, y_i, t_{j_i}) - C(S, x_i, y_i, t_{j_i})]^2 \quad \dots(4)$$

It applies the minimum sum of squares of the difference between the monitored value at different monitoring points and times and the calculated value as the objective function to obtain the optimal solution  $S$  which meets the requirements.

## DIFFERENTIAL EVOLUTION (DE) ALGORITHM

### Brief Introduction of DE Algorithm

DE algorithm, is a novel random optimization method based on population. DE algorithm is effective in solving the identification problems of discontinuous coefficients, including the inverse problems of the source item identification, parameter identification, initial conditions identification, etc., in the inverse problem of partial differential equations.

**Inversion of instantaneous pollution sources in wide and shallow rivers:** Suppose that the research area is a rectangular river 3000 m in length and 150 m in width, as shown in Fig. 1. The flow speeds are  $u_x = 0.65\text{m/s}$  and  $u_y = 0.03\text{m/s}$  and dispersion coefficients are  $D_x = 4.0$  and  $D_y = 0.5$ . The pollutants with mass of 2000 kg are instantly discharged at the location of  $x_m = 500\text{m}$  and  $y_m = 50\text{m}$ . The emission time is recorded as  $t = 0$ . The degradation coefficient of the pollutants is  $K = 0.01\text{d}^{-1}$ .

To inverse the source intensity and location of pollution sources, monitoring points were set in the downstream of the accident area, to monitor the change of pollutants concentration. Typical cases were studied by setting two

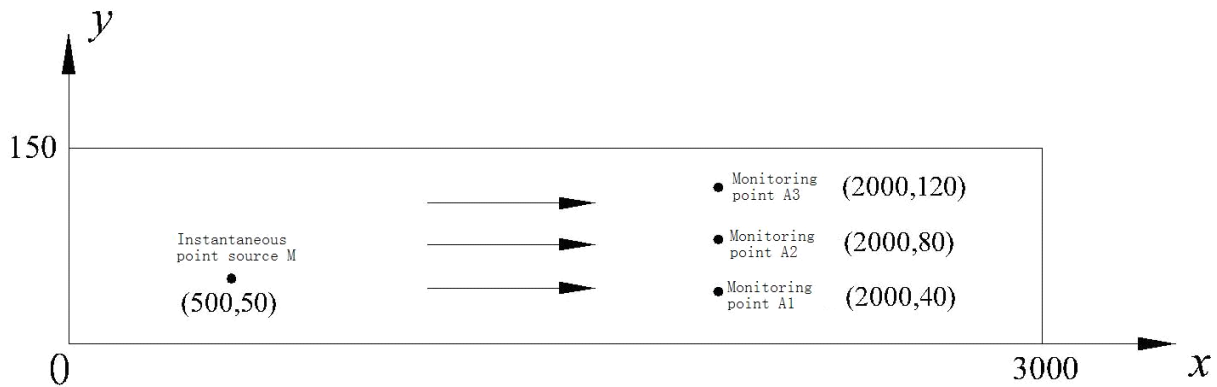


Fig. 1 The schematic diagram of inversing pollution sources in wide and shallow rivers.

work conditions. The experiments provided reference for the identification of pollution sources in practical application.

**Work condition 1:** At 1200 s after the occurrence of the accident, the pollutant concentration was monitored at  $A_1(2000,40)$ ,  $A_2(2000,80)$  and  $A_3(2000,120)$  simultaneously. The monitoring was performed every 500 s and stopped at the 3700 s after the accident. The monitoring was conducted for 6 times at each point.

**Work condition 2:** At 1000 s after the occurrence of the accident, pollutant concentration was monitored at points  $B_1(1500,50)$  and  $B_2(1500,100)$  until 2000 s after the accident. Then after another 1000 s, the pollutants concentration was monitored at  $B_3(2500,50)$  and  $B_4(2500,100)$  and the monitoring stopped at the 4000 s after the accident. Each point was monitored every 500 s for 3 times.

The pollutants concentration  $C_{obs}'(x_i, y_i, t_j)$  ( $i = 1, 2, \dots, N_j, K, N$  - are the total numbers of monitoring points and monitoring frequency at  $i^{th}$  monitoring point) at the  $i^{th}$  monitoring point and  $j^{th}$  monitoring series was calculated using the analytic solution (2) of the forward problem. This pollutant concentration was used as the theoretical value of the monitoring information. Meanwhile, the monitoring error

was taken into account and a disturbance was added to the theoretical value. After superposing the monitoring error, the monitoring value was:

$$C_{obs}(x_i, y_i, t_j) = C_{obs}'(x_i, y_i, t_j) + e \cdot \omega_{i,j} \cdot C_{obs}'(x_i, y_i, t_j) \dots(5)$$

Where  $C_{obs}'(x_i, y_i, t_j)$  is the theoretical monitoring value of the pollutants concentration calculated using the analytic solution of the forward problem;  $C_{obs}(x_i, y_i, t_j)$  is the monitoring value of pollutants concentration considering monitoring error;  $e$  is monitoring error level (according to relevant data and the monitoring accuracy of current monitoring technology, the error was  $e = 0.05$ ); and  $\omega_{i,j}$  is the random number which is in line with the standard normal distribution.

The theoretical value  $C_{obs}'(x_i, y_i, t_j)$  and actual value  $C_{obs}(x_i, y_i, t_j)$  of the monitoring information were calculated using the analytic solutions (2) and (5) of the forward problem, respectively. The theoretical and actual values of the first and second work conditions are listed in Table 1 and Table 2.

The intensity and location of the instantaneous pollution sources in the wide and shallow river were inverted using the monitoring information in Table 1 and Table 2. The parameters of DE algorithm were set as follows: Population size was  $N = 20$ , crossover probability was  $P_c = 0.1$ , crossover factor was  $P_m = 0.5$ , and the maximum evolutionary generation was  $\max Gen = 500$ . By employing the DE algorithm program, the inverse results of the first and second work conditions were obtained and demonstrated in Table 3.

Table 3 indicates that the inverse results in the two work conditions are the same when the monitoring information is calculated using the analytic solutions of the forward prob-

Table 1: Theoretical value  $C_{obs}'$  and actual value  $C_{obs}$  of monitoring data in first work condition (Unit:  $g/m^3$ ).

Time $t(s)$	$A_1(2000,40)$		$A_2(2000,80)$		$A_3(2000,120)$	
	$C_{obs}'$	$C_{obs}$	$C_{obs}'$	$C_{obs}$	$C_{obs}'$	$C_{obs}$
1200	0.000	0.000	0.000	0.000	0.000	0.000
1700	0.060	0.058	0.158	0.156	0.162	0.157
2200	9.611	8.989	26.604	25.621	35.588	35.818
2700	1.524	1.576	4.363	4.512	6.906	6.821
3200	0.006	0.006	0.018	0.020	0.032	0.035
3700	0.000	0.000	0.000	0.000	0.000	0.000

Table 2:  $C_{obs}$  and  $C_{obs}$  in second work condition (Unit:  $g/m^3$ ).

Time <i>t</i> (s)	$B_1(1500,50)$		$B_2(1500,100)$		$B_3(2500,50)$		$B_4(2500,100)$	
	$C_{obs}$	$C_{obs}$	$C_{obs}$	$C_{obs}$	$C_{obs}$	$C_{obs}$	$C_{obs}$	$C_{obs}$
1000	0.031	0.039	0.039	0.041				
1500	32.034	29.428	62.394	62.845				
2000	1.125	1.084	2.70	2.657				
3000					6.839	7.274	20.205	19.861
3500					1.215	1.223	3.811	4.003
4000					0.011	0.011	0.037	0.034

Table 3: The inverse result of intensity and location of pollution sources in first and second work condition.

	The first work condition			The second work condition		
	<i>M</i> (kg)	$X_m$ (m)	$Y_m$ (m)	<i>M</i> (kg)	$X_m$ (m)	$Y_m$ (m)
Data without errors	2000.00	500.000	50.000	2000.000	500.000	50.000
Data with errors	2014.566	498.755	52.169	2016.823	497.008	52.485
actual value	2000	500	50	2000	500	50

lem without error; when added an error on the monitoring information, though there are errors in the inverse results, the errors are small enough, and the inverse results correctly reflects the emission process of the pollution sources. Therefore, though the monitoring schemes are different in the two work conditions, the intensity and location of the pollution sources are successfully inversed using DE algorithm and provided technical support for the identification of pollution sources.

**RESULTS AND DISCUSSION**

**Identification of factors influencing the inverse accuracy of the intensity of pollution sources:** The monitoring scheme (i.e., the numbers of monitoring points and monitoring frequency) and monitoring accuracy, pose great influence on the inverse accuracy. The influences of monitoring points, frequency and accuracy on inverse accuracy have been quantitatively analysed.

**Influence of the number of monitoring points on the inverse accuracy:** To independently analyse the influence of different monitoring points on the inverse accuracy, the scene analysis method was applied to set three work conditions, that there are respectively one, two and three monitoring points with same monitoring time and monitoring frequency. By carrying out numerical experiments, the influence was analysed quantitatively. The monitoring information of water quality in the three work conditions were calculated using the analytic solutions of the forward problem. To simplify the process, there was no disturbance added. The inverse accuracy of the source intensity is quantita-

tively described using  $E_M$ , and the inverse accuracy of the location was expressed using the absolute errors  $E_x$  and  $E_y$  quantitatively, as shown in equations (6), (7) and (8).

$$E_M = \frac{|M_{estimated} - M_{true}|}{M_{true}} \dots(6)$$

$$E_x = |x_{estimated} - x_{true}| \dots(7)$$

$$E_y = |y_{estimated} - y_{true}| \dots(8)$$

Where  $M_{estimated}$ ,  $x_{estimated}$ , and  $y_{estimated}$  are the calculated values of the intensity *M*, and the coordinates of  $x_m$  and  $y_m$  of the pollution sources, respectively; and  $M_{true}$ ,  $x_{true}$ , and  $y_{true}$  are the exact values of the intensity *M* and the coordinates of  $x_m$  and  $y_m$ .

The inversion is conducted using the DE algorithm program and the results are listed in Table 4.

The inverse results in Table 5 state that when there is a monitoring point, the inverse results present large error and fail to reflect the real emission process of the pollution sources; when there are two or more than two monitoring points, the inverse results are very close to the real situation. Therefore, at least two monitoring points need to be set.

**Influence of monitoring frequency on the inverse accuracy:** In order to study the independent influence of monitoring frequency on the inverse accuracy of source intensity, scene analysis method was employed to set 5 work conditions. The work conditions have same monitoring

Table 4: The inverse results of different monitoring points.

Monitoring sites	$M(\text{kg})$	$E_M$	$X_m(\text{m})$	$E_x$	$Y_m(\text{m})$	$E_y$
$A_2$	5058.872	1.529	497.770	0.004	87.061	0.741
$A_1$ and $A_2$	2000.079	0.000	499.997	0.000	50.001	0.000
$A_1, A_2$ and $A_3$	2000.000	0.000	500.000	0.001	50.000	0.000

Table 5: The inverse results based on different monitoring frequency.

Monitoring frequencies of each monitoring sites	$M(\text{kg})$	$E_M$	$X_m(\text{m})$	$E_x$	$Y_m(\text{m})$	$E_y$
2	13020.245	5.510	562.950	0.126	49.029	0.019
3	4784.767	1.392	245.867	0.508	49.241	0.015
4	2026.784	0.013	498.889	0.002	49.988	0.000
5	1998.766	0.001	499.947	0.000	49.976	0.000
6	2000.000	0.000	500.000	0.000	50.000	0.000

points [ $A_1(2000,40)$  and  $A_2(2000,80)$ ] and different monitoring frequency (2, 3, 4, 5, and 6 times respectively, corresponding to time intervals of 2400 s, 1200 s, 800 s, 600 s, and 480 s). The monitoring was conducted during the period 1200 s to 3600 s after the occurrence of the accident. Numerical experiments were performed to quantitatively analyse the monitoring frequency on the inverse accuracy of the intensity of instantaneous pollution sources. The monitoring information of the water quality of the 5 work conditions were obtained directly from the analytic solutions of the forward problem. To simplify the calculation, there was no disturbance added.

The pollution sources in different work conditions is inverted using DE algorithm and the results are illustrated in Table 5.

The results indicate that, when each point is monitored for less than 4 times, the inverse results show a large error and can not reflect the emission process of the pollution sources; when monitored for 4 times, the inverse results have a slight error and can basically represent the emission process; when monitored more than 4 times, the inverse results are accurate and reliable. In general, with fixed monitoring points, the more times the monitoring carried out, the higher the inverse accuracy is. Therefore, the monitoring frequency has to be increased in the formulation of monitoring scheme to ensure the reliability of the inverse results.

## CONCLUSIONS

Aimed at the problems in the inversion of the intensity and location of instantaneous pollution sources in wide and

shallow rivers in the water quality forecast, the research transformed the inverse problems to the optimum control problems of parameter system. A DE algorithm, which simplified the inversion, was employed. Numerical experiments proved that the analysis of the inverse problems based on DE algorithm was effective and feasible. To improve the analysis accuracy, quantitative analysis was carried out on the monitoring points and monitoring frequency. The results indicate that at least two monitoring points are needed and the monitoring frequency has to be improved. All these are effective in improving the inverse accuracy and ensuring the reliability of inverse results.

## REFERENCES

- Araujo, Susana, Henriques, I.S. and Leandro, S.M. 2014. Gulls identified as major source of fecal pollution in coastal waters: A microbial sourcetracking study. *Science of the Total Environment*, 470: 84-91.
- Han, L.X. 2001. Inverse problem on amount of pollutant into natural channel. *Advances in Water Science*, 12(1): 39-44.
- Han, L.X., Jiang, L.H and Zhu, D.S. 2001. Inverse problem on boundary condition and pollutant source in water quality model of river network. *Journal of Hohai University (Natural Sciences)*, 29(5): 23-26.
- Jin, Z.Q. and Zhou, Z.F. 1997. *The Inverse Problem on Hydraulics*. Hohai University Press, pp. 116-118.
- Jin, Z.Q. and Chen, X.Q. 1992. Numerical solution to an inverse problem of source control for convection-diffusion equations by PST-Optimization. *Journal of Hohai University (Natural Sciences)*, 20(2): 1-8.
- Liu, X.D., Yao, Q., Xue, H.Q., Chu, K.J. and Hu, J. 2009. Advance in inverse problems of environmental hydraulic. *Advance in Water Science*, 20(6): 885-893.
- Min, T., Ma, X.W., Feng, M.Q. and Gao, Z.Q. 2008. A parameter identification inverse problem of the convection diffusion equa-

- tion with more contaminant source. *Journal of Taiyuan University of Technology*, 39(6): 564-567.
- Min, T., Zhou, X.D., Zhou, S.M. and Feng, M.Q. 2009. Genetic algorithm to an inverse problem of source term identification for convection-diffusion equation. *Journal of Hydrodynamics*, 19(4): 520-524.
- Plummer, J.D. and Long, S.C. 2009. Identifying sources of surface water pollution: A toolbox approach. *Journal American Water Works Association*, 101(9): 75-82.