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# **Fitting Probability Distributions and Statistical Trend Analysis of Rainfall of Agro-climatic Zone of West Bengal**

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# **ABSTRACT**

This research aimed to identify the most appropriate probability distribution for modeling average monthly rainfall in the agro-climatic zones of West Bengal and to detect any trends in this data. The study utilized historical rainfall data spanning 51 years (1970-2020) obtained from the IMD in Pune. To determine the best-fitting distribution and assess trends, 23 different probability distributions were employed, with the Mann-Kendall test and Sen's slope estimator used for trend analysis. Goodness-of-fit tests, including the Kolmogorov-Smirnov, Anderson-Darling, and Chi-square tests, were employed to determine the most suitable distribution. The findings indicated that the Generalized Extreme Value, Gamma, and Lognormal (3-parameter) distributions were the best fits for two specific districts. The monthly rainfall distributions can be effectively used for predicting future monthly rainfall events in the region. The Mann-Kendall test revealed an increasing trend in rainfall for Kalimpong and Nadia Districts and a decreasing trend for Malda District.

# **INTRODUCTION**

As climate change faces unprecedented changes, the stability of atmospheric conditions that administer rainfall is being disrupted, leading to a reflective influence on rainfall patterns, distribution, intensity, and frequency. The production and policy-induced abatement operations of the agricultural economy are highly affected by the impact of climate change (Mandal et al. 2013). One of the most obvious impacts of climate change on rainfall is the increase in extreme weather events and the effects of cropping systems and production. Intense and long-lasting periods of rainfall, often accompanied by harsh storms and flooding, have become frequent phenomena in many regions. Moreover, in India-like countries, the agriculture and allied sectors are highly dependent on the monsoon rains that occur between June to mid-October (Sathish et al. 2017). The active monsoon period rainfall is vital for irrigating crops, restocking water reservoirs, and sustaining groundwater levels. In many regions where irrigation infrastructure is limited, the timely arrival and distribution of monsoon rainfall is a vital source of agricultural productivity. Only adequate rainfall can ensure soil moisture balance which is essential for crop growth, development, and yield. Increasingly erratic, unpredictable monsoons coupled with extended dry spells disrupt agricultural planning, constrain crop growth, and increase the risk of pests and disease attacks. Thus, a climate change-induced hazard in rainfall

patterns is a significant challenge to agriculture (Dastidar et al. 2010).

The distinctive characteristics of West Bengal confine the sub-Himalayan in the north and coastal region in the south which makes the state disparate rainfall and cropping pattern in the agro-climatic zones. Hence, the state embraces the six agro-climatic zones namely Northern hilly, Terai-Teesta alluvial, Gangetic alluvial, Vindhyan alluvial, Undulating Red and Laterite and Coastal saline zone with their annual rainfall ranging from 1700 to 3550 mm (Mondal 2021). The state is at the forefront of inland fish, rice, and jute production and the second largest producer of potatoes owing to having expanded alluvial plains in Gangetic and Vindhyan alluvial zones and river basins namely the Ganga at the area of 81%, the Brahmaputra (12%) and Subarnarekha (Bandyopadhyay et al. 2014).

Studied probability analysis of daily maximum rainfall data for 37 years in six distinct locations of West Bengal to find out the best distribution model that could represent rainfall extremities, the sum of rank results revealed that Log Pearson type 3 distributions were best fit for three geographical places, namely Kharagpur, Bolpur, and Balurghat, Gumble distribution and Log logistic for Kolkata and Darjeeling, respectively (Basak et al. 2019). The distribution of Log-Logistic, Generalized Extreme Value, Pearson 5, Log-Pearson 3, 3-parameter Dagum, 4-parameter Generalized Gamma, and 3-parameter Generalized Gamma



Fig. 1: Selected observatory in agro-climatic zones of West Bengal.

turned out to be best fit for monthly rainfall time series data from 1901 to 2013 in thirteen districts of Gangetic West Bengal (Pal & Majumdar 2015), Generalized Pareto distribution as best daily seasonal rainfall data from June to September in Nanded district (Alam et al. 2018), the best-fitted distribution for daily maximum rainfall in Karnataka is selected according to goodness of fit criteria mainly Kolmogorov-Smirnov, Anderson Darling and Chisquared (Bhavyashree & Bhattacharyya 2018). Existing monotonically increasing and decreasing trends by Mann-Kendal Test and Sen's slope estimator test to assess their strength (Gowthaman et al. 2023). Due to climate change long-term upward and downward significant rainfall trends in West Kalimantan (Aditya et al. 2021) and rainfall trend analysis using Mann-Kendal and Sen's slope estimator test in Vamanapuram River basin, South Kerala (Brema 2018). Thus, the present study emphasizes the identification of the best-fitted probability distribution to model rainfall amounts by comparing different probability distributions for agroclimatic regions of West Bengal. Moreover, patterns or trends of climatic variables (Rainfall) have been examined.

## **MATERIALS AND METHODS**

Rainfall data for six districts, namely Kalimpong, Malda,

Coochbehar, Nadia, Birbhum, and Sagar Island, located in different agro-climatic zones, was collected from the Indian Meteorological Department (IMD) in Pune. The dataset covers a time frame of 51 years (1970 -2020). This data pertains to seasonal rainfall and specifically represents the average precipitation occurring during the active monsoon period, spanning from June to October. Fig. 1 provides a visual representation of the selected district within each corresponding agro-climatic zone.

## **Fitting Probability Distributions**

The study involved the use of 23 different continuous probability distribution models to assess their goodness of fit. Three statistical tests, namely the Kolmogorov-Smirnov, Anderson-darling, and Chi-Squared tests, were applied to evaluate the suitability of these selected distributions for seasonal rainfall data recorded during the monsoon period from June to October. Each GOF test generated a test statistic, which was then tested at a significance level of  $\alpha$ =0.05. Subsequently, for each of the three GOF tests, individual rankings were assigned to all the distributions based on their test statistic values. A ranking-based scoring technique was employed to identify the most appropriate distribution for monthly rainfall data, following the methodology outlined







Table cont....



by Sharma & Singh (2010). Detailed information on the 23 continuous probability density functions and their respective parameters can be found in Table 1.

## **Kolmogorov-Smirnov Test**

Let  $(x_1, x_2, ..., x_n)$  be rainfall data with CDF F(x) from the continuous distribution. The difference between the theoretical and empirical cumulative distribution functions gives the test statistic  $(D_n)$  as

$$
D_n = \max_{1 \le i \le n} \left( F(x_i) - \frac{i-1}{n}, \frac{i}{n} - F(x_i) \right)
$$

#### **Anderson-Darling Test**

Comparing the fit of actual and expected actual Cumulative distribution function with giving weightage to tail characteristics of the distribution (Anderson & Darling 1954). It is defined as



$$
A^{2} = -n - \frac{1}{n} \sum_{i=1}^{n} (2_{i} - 1) [lnF(x_{i}) + ln(1 - F(X_{n-i+1}))]
$$

#### **Chi-Squared Test**

It is the non-parametric test that is used to test whether there is any difference between observed  $(O_i)$  and expected  $(E_i)$ frequency. The test statistic as:

$$
\chi^{2} = \sum_{i=1}^{n} \frac{(O_{i} - E_{i})^{2}}{E_{i}}
$$

### **TREND ANALYSIS**

#### **Mann-Kendall Test**

The linear regression trend analysis needs the distributionfree (non-parametric) test when the estimated slope of linear regression is different from zero. Which can be accomplished by the Mann-Kendall (M-K) test (Mann 1945). Hence, it asses statistically the monotonic upward (downward) trend in time series rainfall data. In addition, the M-K test is not influenced by the outliers since it depends on positive and negative signs. The strength of the trend depends upon the magnitude, sample size, and variations of data series. The M-K test statistic is equated as:

$$
S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn (X_j - X_i)
$$

The trend test is applied to  $X_i$  data values (i=1, 2, ...n) and  $X_j$  (j=i+1, 2,...n). The data values of  $X_i$  are used as a reference point to compare with the data values of  $X_j$  which is given as:

$$
sgn(X_j - X_i) = \begin{cases} -1 \text{ if } (X_j - X_i) < 0 \\ 0 \text{ if } (X_j - X_i) = 0 \\ 1 \text{ if } (X_j - X_i) > 0 \end{cases}
$$

The above statistic represents the number of positive differences minus the number of negative differences for all the differences considered. The normal distribution with mean and variance  $(S=0)$  test is conducted, when the sample is large (>10) and the standard normal Z-statistic is given as

$$
Z = \begin{cases} \frac{S-1}{\sqrt{var(S)}} & \text{if } S > 0\\ 0 & \text{if } S = 0\\ \frac{S+1}{\sqrt{var(S)}} & \text{if } S < 0 \end{cases}
$$
 where,  $var(S)$ 

$$
-F(X_{n-i+1})\bigg] = \frac{n(n-1)(2n+5) - \sum_{i=1}^{n} t_i(t_i-1)(2t_i+5)}{18}
$$

Where  $n$  - number of tied groups and  $t_i$  - number of data points in the i<sup>th</sup> tied groups. The upward and downward trend is interpreted based on the positive and negative values of Z statistic respectively.

#### **Sen's Slope Estimator Test**

Although the trend is identified by the M-K test, the magnitude of the trend is determined by a Theil-Sen slope or Sen's Slope Estimator (SSE), which is a robust method against outliers to estimate the slope of the trend (Sen 1968).

$$
T_i = \frac{x_j - x_k}{j - k}
$$

Where  $x_j$  and  $x_k$  are date values at the time *j* and *k* respectively. The median of these n values of  $T_i$  is represented as Sen's estimator of slope which is given as:

$$
Q_i = \begin{cases} \n\frac{T_{n+1}}{2} & \text{for } n \text{ is odd} \\ \n\frac{T_n + T_{n+1}}{2} & \text{for } n \text{ is even} \n\end{cases}
$$

A positive value of  $Q_i$  indicates an upward or increasing trend and *vice versa*.

### **RESULTS AND DISCUSSION**

Table 2 provides a summary of statistics for a specific district within the agro-climatic zones of West Bengal. The results point out that the Kalimpong district in the Hill and Terai agro-climatic zones experiences the highest levels of rainfall throughout West Bengal. It receives significantly more rainfall compared to other zones, with Coochbehar coming next. Conversely, the Birbhum district has consistently received the lowest amount of rainfall over the years.

where,  $var(S)$  and a higher intention of a more even distribution with fewer extreme values. The skewness and kurtosis values 18 collectively suggest that the data in these districts do not In terms of statistical characteristics, the Nadia district displays positive skewness and kurtosis values, suggesting that most of the rainfall events during this period have relatively lower intensity, with only occasional instances of heavy rainfall. For the Kalimpong district, both skewness and kurtosis values are negative, indicating a prevalence of rainfall events with lower intensity and infrequent heavy rainfall occurrences. In contrast, Birbhum exhibits the lowest kurtosis value, implying a flatter peak near the mean and a higher likelihood of a more even distribution with follow a normal distribution.

<b>Districts</b>	Minimum	Maximum	Mean	<b>SD</b>	CV	<b>Skew</b>	Kurtosis
Kalimpong	1329.45	4518.30	2914.34	797.37	27.36	$-0.01$	$-0.73$
Coochbehar	1709.61	4715.28	2670.08	587.87	22.02	0.89	1.36
Malda	401.00	2034.54	1108.09	305.42	27.56	0.14	1.16
Nadia	711.17	2370.07	1208.71	284.44	23.53	. 46	4.24
Birbhum	593.54	1820.34	1187.34	250.61	21.11	0.24	0.21
Sagar island	910.66	2794.20	1517.51	340.69	22.45	1.12	2.63

Table 2: De**s**criptive statistics for agro-climatic zones.

Table 3: Score-wise best fitted probability distribution with parameter estimates.



Table 4: Results of M-K and Sen's slope estimator test on agro-climatic zones.



## **Distribution Fitting**

The study involved fitting average monthly rainfall data from six different districts in various agro-climatic zones of West Bengal to 23 different continuous probability distributions. For each district, three test statistics were computed using the Kolmogorov-Smirnov, Anderson-darling, and Chisquare goodness of fit tests. Each distribution was ranked separately for each test, and distributions that failed to fit the data received no rank. Since different distributions ranked differently in each goodness of fit test, it was challenging to determine a single best-fit distribution for each district. Consequently, a scoring method was employed as described in the methodology. Scores were assigned to each distribution for all three tests, and the final score was calculated by summing these three scores. The distribution with the highest total score was considered the best fit for the respective district.

The analysis of goodness-of-fit test results revealed that, in many cases, there was minimal difference between various distributions for each district. Furthermore, no single distribution consistently ranked as the best fit across all locations. However, the Generalized Extreme Value, Gamma, and Lognormal (3-parameter) distributions emerged as the most suitable choices for the two districts. Specifically, the Generalized Extreme Value distribution was found to be the best fit for Kalimpong and Nadia districts, the Gamma Distribution for Coochbehar and Birbhum districts, and the Lognormal (3-Parameter) distribution for Malda and Sagar Island districts. Table 3 presents the best-fitted probability distribution for each district, along with the parameter estimates. In summary, the monthly rainfall distribution in West Bengal appears to be positively skewed, and the Gamma and Log-Normal distributions can be effectively used for predicting future monthly rainfall events in the region.

## **Trend Analysis**

The study examined the average monthly rainfall data for six districts within the agro-climatic zones of West Bengal using the Mann-Kendall test and Sen's slope estimator. Table 4 presents the M-K test statistics and associated p-values, and





Fig. 2: Trend graph of rainfall for the period from 1970 to 2020.

Fig. 2 depicts the trend graph of rainfall for these districts. In this context, p-values less than 0.05 were considered significant, indicating a rejection of the null hypothesis, which assumes no trend in the data. The results revealed that the Kalimpong and Nadia districts had significant p-values of 0.42 and 0.34, respectively, with positive Kendal's Tau values, suggesting an increasing trend. In contrast, the p-value for Malda district was below the significance level at 0.26, accompanied by a negative Kendal's Tau value, indicating a decreasing trend.

Conversely, for Coochbehar, Birbhum, and Sagar Island districts, the p-values were 0.56, 0.62, and 0.64, respectively, exceeding the significance level of 0.05, indicating the absence of a significant trend in their data. The findings from the Sen's slope test supported those of the M-K test, and the Sen's slope values were provided in Table 4. It's worth noting that Sen's slope values were also calculated for districts with no discernible trend. This is because the M-K test considers the hypothesis above the 5% significance level, allowing for the possibility of a trend's existence beyond this threshold. The Sen's slope values for Kalimpong and Nadia

indicated positive slopes of 33.19 and 29.64, respectively, while the remaining districts exhibited negative slopes over the years. These results contributed to the assessment of the average monthly rainfall levels in the selected districts of West Bengal.

#### **CONCLUSIONS**

A systematic evaluation approach was employed to determine the optimal probability distribution for modeling monthly rainfall data in six different districts of West Bengal. The study utilized the Mann-Kendall test and Sen's Slope estimation techniques to identify any monotonic trends in the data. Notably, the Generalized Extreme Value, Gamma, and Lognormal (3-parameter) distributions were found to be the most suitable choices for two of the districts each. According to the Mann-Kendall test results, Kalimpong and Nadia Districts exhibited an increasing trend in rainfall, while Malda District showed a decreasing trend. The ability to identify both the distribution and trend of rainfall has significant implications in various fields, including agriculture, hydrology, and climate research.

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