



# Numerical Investigation of Heterogeneous Nucleation of Supersaturated Water Vapour on Coal-fired PM<sub>10</sub>

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## ABSTRACT

The kinetic model is adopted to describe heterogeneous nucleation of supersaturated water vapour on coal-fired PM<sub>10</sub>. To verify the accuracy of the kinetic model, it is compared with the Fletcher model and experimental data. Additionally, the comparison for condensation and evaporation coefficients and the relative importance of two diffusion condensation mechanisms are systematically analysed during embryo growth process. Furthermore, the influence of vapour temperature on nucleation rate and critical supersaturation for coal-fired PM<sub>10</sub> are researched. The results show that the predicted critical supersaturation for Kinetic model is far closer to the experimental data compared with the Fletcher model. What is more, once the embryo radius  $r_n$  reaches to the critical embryo radius  $r^*$ , it can grow spontaneously, and the indirect surface diffusion mechanism is more important than the direct addition mechanism to embryo growth, the value of RTO is always above 100. It is also found that increase in the vapour temperature is conducive to nucleation process, which can increase the nucleation rate and decrease the barrier of nucleation free energy.

## INTRODUCTION

The frequent haze weather has attracted people's extensive attention to the particulate matters in recent years (Siddharth et al. 2008). Coal-fired power plants are one of the main sources for particulate matter in the atmosphere environment (Liu et al. 2016). These particles can not only cause serious environmental problems but also pose a serious threat to people's health. Particles with the diameter less than 10 microns can enter the respiratory system of the human body, causing damage to the heart and lung functions, resulting in arrhythmia, myocardial infarction, heart failure, atherosclerosis and other diseases (Bentayeb et al. 2012).

However, conventional particle abatement devices such as cyclone separator, water scrubber, fabric filters, electrostatic precipitator and so on have less effective in collecting submicron particles (Hu et al. 2018, Zhou et al. 2016). Therefore, to find a valid preconditioning method to improve the removal efficiency for fine particles has become an urgent problem. Heterogeneous condensation as a promising preconditioning technique has been widely used in the traditional particle separation processes (Heidenreich et al. 1995, Hao et al. 2016). At present, a large number of experimental and theoretical researches focus on the removal efficiency for fine particles by means of vapor heterogeneous condensation technology, while ignoring the heterogeneous nucleation process at the initial stage of heterogeneous

condensation of water vapor (Hao et al. 2016, Zhou et al. 2016, Wen et al. 2014). Fan et al. (2013) pointed out that the whole process for heterogeneous condensation include three stages: nucleation, transition and growth stage. The heterogeneous nucleation stage plays an important role in the process of steam heterogeneous condensation, so it is of great significance to study the heterogeneous nucleation characteristics for supersaturated steam on particle surface.

Studies on heterogeneous nucleation have a long history. Fletcher (1958) was the first to study heterogeneous nucleation of vapor on insoluble spherical particles, which contained the effect of particle size. Fan et al. (2007) numerically investigated the water vapor (with or without wetting agents) on PM<sub>2.5</sub> heterogeneous nucleation behaviour according to Fletcher's classical nucleation theory. However, Chen et al. (2000) revealed that the prediction of critical supersaturation for Fletcher classical nucleation theory was flawed, which was too high compared with experimental data. In recent years, the kinetic model has been widely used to investigate heterogeneous nucleation of supersaturated vapor on particles. Luo et al. (2014) derived the growth rate for the cap shaped embryo formed on the particle surface based on the kinetic nucleation model. Fan et al. (2015) put forward to a modified expression for the steady-state heterogeneous nucleation rate based on the kinetic model and validated the advantage of the expression. All these

researches indicate that the kinetic model is a valid tool to investigate the phenomenon for heterogeneous nucleation of supersaturated water vapor on particle.

In this paper, we apply the kinetic model (Luo et al. 2014), in which three factors are taken in account, including the effect of line tension, two mechanisms of embryo growth and coal-fired PM<sub>10</sub> surface wettability, to research the heterogeneous nucleation characteristics of supersaturated vapor on spherical coal-fired PM<sub>10</sub>. In addition, the effects of operational parameters including temperature and supersaturation of vapor on heterogeneous nucleation parameters are also investigated.

**Physical Model**

Fig.1 shows the growth process of embryo formed on coal-fired PM<sub>10</sub> surface. The embryo can grow via the condensation of water vapor molecules direct diffusion from the vapor phase and the indirect diffusion for particle surface absorbed water at the rate of  $C_n(\alpha)$  and  $C_n(\beta)$ , respectively. Meanwhile, condensation and evaporation occur simultaneously, the embryo droplet could also shrink by evaporation of water molecules to both the vapor phase and

particle surface at the rate of  $E_n(\alpha)$  and  $E_n(\beta)$ , respectively.

Fig. 2. shows the process of the number changes for embryo. The number of embryos formed on particle surface per unit area is defined as  $f_n$  and the change rate of  $f_n$  is written as:

$$\frac{df_n}{dt} = C_{n-1}f_{n-1} - (C_n + E_n)f_n + E_{n+1}f_{n+1} \quad \dots(1)$$

Where,  $C_n$  and  $E_n$  denote the coefficients of condensation and evaporation for embryo droplet, respectively.

When we take into account the effect of line tension in kinetic nucleation model, the free energy for forming an embryo on coal-fired PM<sub>10</sub> is given by (Luo et al. 2014).

$$\Delta G = \Delta G_v V + s_{lv} S_{lv} + (s_{sl} - s_{sv}) S_{sl} + \tau L_{scv} \quad \dots(2)$$

Where,  $V$  is the volume of embryo droplet ( $m^3$ ),  $\Delta G_v$  represents the free energy difference between vapor phase and liquid phase unit volume ( $J \cdot m^{-3}$ ),  $\sigma_{ij}$  ( $N \cdot m^{-1}$ ) is the surface free energy between phase  $i$  and  $j$ , and  $S_{ij}$  is the surface area of the interface ( $m^2$ ).  $\tau$  (N) is the line tension at the triple phase boundary between embryo, substrate, and vapor phase and  $L_{scv}$  is the total length for the three-phase contact line (m).

From Fig. 1, the geometric parameters are written as:

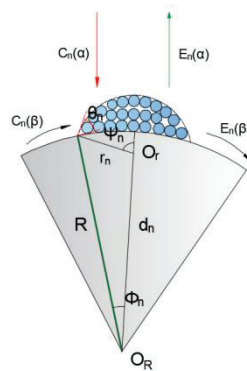


Fig. 1: Schematic diagram for the growth process of embryo.

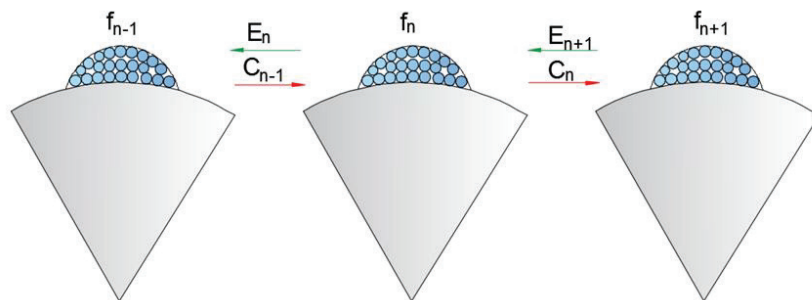


Fig. 2: Kinetic process for the change rate of the embryo number.

$$V = \frac{1}{3} \rho r_n^3 (2 - 3 \cos \gamma_n + \cos \gamma_n^3) - \frac{1}{3} \rho R^3 (2 - 3 \cos \Phi_n - \cos \Phi_n^3) \quad \dots(3)$$

$$S_{lv} = 2 \rho r_n^2 (1 - \cos \gamma_n) \quad \dots(4)$$

$$S_{sl} = 2 \rho R^2 (1 - \cos \Phi_n) \quad \dots(5)$$

$$L_{scv} = 2 \rho r_n \sin \gamma_n \quad \dots(6)$$

$$d = \sqrt{r_n^2 + R^2 - 2mr_nR} \quad \dots(7)$$

$$\cos \gamma_n = \frac{mR - r_n}{d} \quad \dots(8)$$

$$\cos \Phi_n = \frac{R - r_n m}{d} \quad \dots(9)$$

Where,  $r_n$  represents embryo radius (m),  $R$  is the radius of coal-fired particle (m),  $d$  is the distance between the centre of embryo droplet and that of particle (m),  $m$  is the cosine of microscopic contact angle  $\alpha_n$ .

The microscopic contact angle  $\alpha_n$  is a function of line tension  $t$  and macroscopic contact  $\alpha_\infty$  (Hienola et al. 2007).

$$\cos \alpha_\infty = \cos \alpha_n + \frac{t(R - r \cos \alpha_n)}{S_{lv} r_n R \sin \alpha_n} \quad \dots(10)$$

The radius of the critical embryo is:

$$r_n^* = \frac{2 S_{lv} V_{wm}}{k_B T \ln S} \quad \dots(11)$$

Where,  $k_B$  is the Boltzmann constant ( $1.38 \times 10^{-23} \text{J}\cdot\text{K}^{-1}$ ),  $T$  (K) is the temperature of water vapour,  $V_{wm}$  is the volume of a water molecule ( $\text{m}^3$ ),  $S$  is water vapour supersaturation, which is the ratio of the ambient vapour pressure to saturation vapour pressure.

The heterogeneous nucleation free energy barrier corresponds to the maximum value of  $\Delta G$ , which is the formation energy for the critical embryo (Hienola et al. 2007).

$$\Delta G^* = \frac{2 \rho r_n^{*2} S_{lv}}{3} f(m, x) + 2 \rho R t \left( \frac{1 - \cos \Phi_n}{\sin \Phi_n} \right) \quad \dots(12)$$

Where,  $f(m, x)$  is the geometrical factor (Fan et al. 2007).

The expression of condensation coefficient in the kinetic model for embryo is (Xu et al. 2017):

$$C_n = C_n(a) + C_n(b) \quad \dots(13)$$

$$C_n(a) = a_n \frac{P_v}{\sqrt{2 \rho m_{wm} k_B T}} S_{cv}(r_n) \quad \dots(14)$$

$$C_n(b) = N_1 u d \exp\left(-\frac{\Delta G_{diff}}{k_B T}\right) L_{cvs}(r_n) \quad \dots(15)$$

$$N_1 = \frac{P_v}{u \sqrt{2 \rho m_{wm} k_B T}} \exp\left(\frac{\Delta G_{des}}{k_B T}\right) \quad \dots(16)$$

Where,  $P_v$  is the ambient pressure of water vapor (Pa),  $u$  denotes the vibration frequency of one molecule on the surface ( $\text{s}^{-1}$ ),  $d$  is the mean jump distance for one water molecule (m),  $\Delta G_{diff}$  is the surface diffusion energy for one water molecule (J),  $a_g$  denotes the sticking coefficient of embryo droplet.  $N_1$  is the surface concentration of water molecules.  $\Delta G_{des}$  is the desorption energy for individual water molecule (J).

The explicit expression of evaporation coefficient is written as (Fan et al. 2015):

$$E_n = E_n(a) + E_n(b) \quad \dots(17)$$

$$E_n(a) = \exp\left(\frac{2 S V_{wm}}{k_B T r_n}\right) \frac{P_v^{sat}}{\sqrt{2 \rho m_{wm} k_B T}} S_{cv}(r_n) \quad \dots(18)$$

$$E_n(b) = \exp\left(\frac{2 S V_{wm}}{k_B T r_n}\right) d N_1^{sat} u \exp\left(\frac{\Delta G_{diff}}{k_B T}\right) L_{cvs}(r_n) \quad \dots(19)$$

The heterogeneous nucleation rate is one of crucial parameters to research heterogeneous nucleation process and the expression of nucleation rate is (Fan et al. 2015):

$$J = Z \cdot [C_n^*(a) + C_n^*(b)] \cdot \exp\left(-\frac{\Delta G^*}{k_B T}\right) \quad \dots(20)$$

and

$$Z = \frac{s}{\sqrt{k_B T (2 + \cos \gamma_{n^*})}} \cdot \frac{V_{wm}}{\rho r_n^{*2} (1 - \cos \gamma_{n^*})} \quad \dots(21)$$

Table 1: Parameters for the coefficients of condensation and evaporation in heterogeneous nucleation on coal-fired particle.

Parameter	Symbol	Value
Mean jump distance	$\delta$ (nm)	0.32
Line tension	$\tau$ ( $\text{N}\cdot\text{m}^{-1}$ )	$-1 \times 10^{-11}$
Macroscopic contact angle	$\theta_\infty$ ( $^\circ$ )	40
Surface diffusion energy	$\Delta G_{diff}$ ( $\text{J}\cdot\text{molecule}^{-1}$ )	$2.9 \times 10^{-21}$
Desorption energy	$\Delta G_{des}$ ( $\text{J}\cdot\text{molecule}^{-1}$ )	$2.9 \times 10^{-20}$

Where,  $Z$  is the Zeldovich non-equilibrium factor, the superscript ‘\*’ denotes the parameters for critical embryo.

The parameters that we used in the kinetic model is listed in Table 1 (Fan et al. 2019 & Xu et al. 2017).

## RESULTS AND DISCUSSION

### Comparison with the Experimental Data

When the heterogeneous nucleation rate  $J = 1 \text{ s}^{-1}$ , the supersaturation is defined as critical supersaturation, which is the minimum barrier to form a new phase from vapor phase. Fig. 3 shows the comparison of predicted critical supersaturation for Fletcher model (Fletcher 1956) and kinetic model (Luo et al. 2014) with the experimental data (Chen et al. 2000) in heterogeneous nucleation of water vapor on  $\text{SiO}_2$  particles. It is found that compared with Fletcher model, the predicted critical supersaturation for kinetic model is much closer to the experimental data. Our prediction results are more consistent with the experimental data for small particles. With the increase of particle size, the prediction data for kinetic model are a little larger than the experimental data, which may be accounted by the roughness of particles.

### Contribution of Two Diffusion Mechanisms to Embryo Growth

To investigate the relative importance of the two mechanisms to embryo growth, we define a dimensionless number  $RTO$

as:

$$RTO = C_n(a) / C_n(b) \quad \dots(22)$$

Fig. 4 shows the variations of  $RTO$  with particle diameter and embryo radius. The temperature for supersaturated water vapour is  $40^\circ\text{C}$  (313.15K), the supersaturation is 1.2, and under this condensation the critical embryo radius  $r^*$  is about 5.31nm. It is found from Fig. 4 that when  $r_n < r^*$ , the particle surface diffusion mechanism plays a dominant action in embryo growth and the value of  $RTO$  is always above 100. Additionally, for a given particle diameter, the value of  $RTO$  decreases remarkably with the increasing of embryo radius, which indicates that the mechanism of direct diffusion condensation begins to make more contributions to embryo growth. All these results indicate that the surface diffusion mechanism is more important than the direct addition mechanism when the embryo radius is less than critical embryo radius, and the direct diffusion mechanism plays a significant contribution to the growth of embryo with the increase of embryo size, which is consistent with the conclusion of Fan et al. (2019).

### Coefficients of Evaporation and Condensation

Fig. 5 shows the variation of the coefficients of evaporation and condensation with the radius of embryo under different temperatures. It is found that the coefficients of evaporation and condensation increase nearly linearly with the increase of embryo radius  $r_n$ . When the embryo radius is less than the

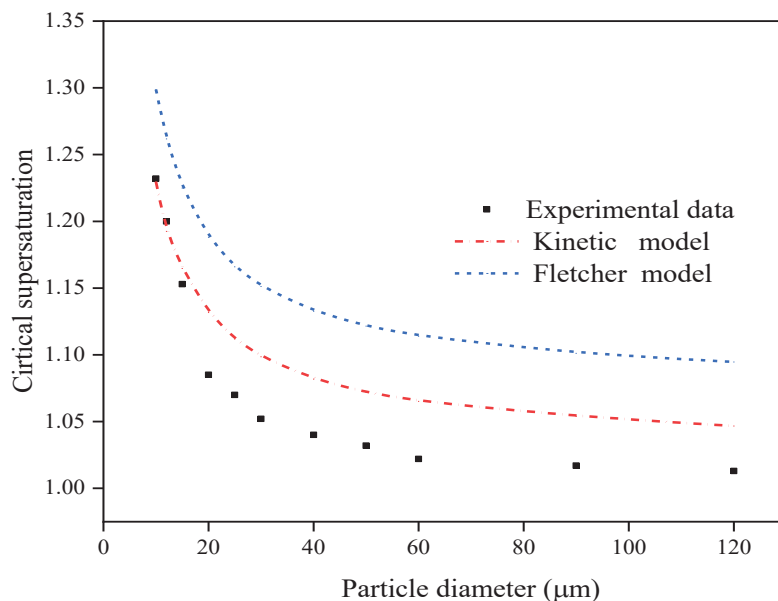


Fig. 3: Comparison with the experimental data.

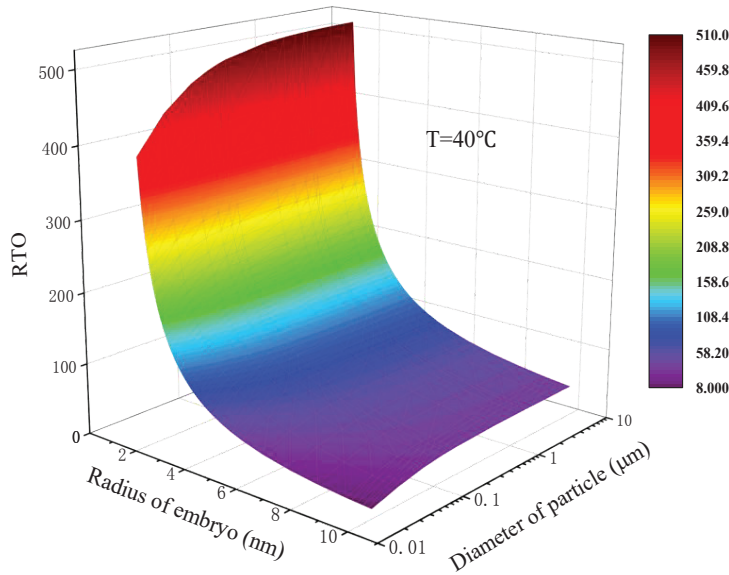


Fig. 4: Variations of RTO with particle diameter and embryo size.

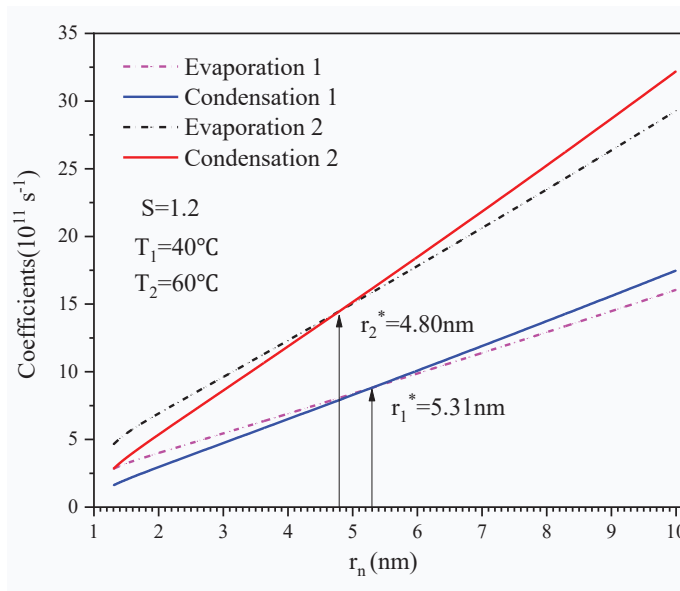


Fig. 5: Variations of evaporation and condensation coefficients with embryo radius.

critical embryo radius  $r_n < r^*$ , the evaporation coefficient is larger than condensation coefficient and the formed embryo tends to disappear. When  $r_n > r^*$ , the evaporation coefficient is less than condensation coefficient and the formed embryo tends to be larger. When  $r_n = r^*$ , the value of evaporation coefficient is equal to condensation coefficient, which means that the embryo is in metastable state and the size of the embryo keeps constant.

Moreover, with the increase of temperature, the value

of evaporation and condensation coefficients are enlarged and the laws between them remain the same at different temperatures, which manifests that the improvement of vapour temperature can accelerate the growth of embryo and benefit the heterogeneous nucleation process.

### Nucleation Rate

Nucleation rate refers to the number of critical embryos formed on coal-fired PM<sub>10</sub> surface unit time. Fig. 6 shows

that the nucleation rate varies with supersaturation at different particle surface. It can be seen from Fig. 6(a) that the nucleation rates increase exponentially with the increase of supersaturation for a given particle diameter. In the case of particles with the diameter of 2.5 microns, the nucleation rate increases from  $6.5 \times 10^5$  to  $6.9 \times 10^{18}$ , when the supersaturation increases from 1.2 to 1.3. This is because the increase of supersaturation can reduce the free energy barrier of nucleation and decrease the size of critical embryo radius. Additionally, with the increase of supersaturation, the number of water vapor molecules around the particles is further increased, and there will be more water vapor molecules condensed on particle unit time, all these reasons can account for the changes of nucleation rate.

Fig. 6(b) shows the variations of nucleation rate with vapor supersaturation under different temperatures. It is found that nucleation rate is increased with temperature when the supersaturation is constant. With the temperature increased from 313.15K to 333.15K, the nucleation rate is increased from  $1.39 \times 10^{25}$  to  $3.45 \times 10^{26}$  under the condition  $S = 1.5$ . This is because the improvement of temperature can dramatically increase the diffusion condensation rates for water vapor molecules and decrease the surface tension of embryo droplet. Furthermore, as the supersaturation to be larger, temperature is not the dominant factor to affect the condensation rate and the increase of temperature is not obvious to accelerate the improvement of nucleation rate.

### Critical Supersaturation

Fig. 7 shows the variation of critical supersaturation with

particle diameter and temperature. It can be seen from Fig. 7; the critical supersaturation decreases remarkably with the increase of particle diameter when the particle diameter is less than  $0.1\mu\text{m}$  at different temperatures. As the particle diameter becomes larger, the trend of decrease for critical supersaturation tends to level off. Additionally, the critical supersaturation decreases with vapour temperature for a given particle, when the temperature rises from 313.15K ( $40^\circ\text{C}$ ) to 333.15K ( $60^\circ\text{C}$ ), and the critical supersaturation for coal-fired particles with a diameter of  $2.5\mu\text{m}$  is reduced from 1.180 to 1.152. This is because the increase of temperature can not only decrease the critical embryo size, but also reduce the barrier of free energy for heterogeneous nucleation, which can also account for the increase of nucleation rate with the increase of temperature. Therefore, the improvement of temperature is conducive to the occurrence of heterogeneous nucleation of supersaturated vapour on coal-fired  $\text{PM}_{10}$  surface.

### CONCLUSIONS

1. The kinetic model, in which three factors are considered including the effect of line tension, two mechanisms of embryo growth and the wettability of coal-fired  $\text{PM}_{10}$  can be used to accurately predict the heterogeneous nucleation process.
2. When the embryo radius is less than critical embryo radius  $r_n < r^*$ , the mechanism of particle surface diffusion plays a dominant action in embryo growth and the value of  $RTO$  is always above 100.
3. The heterogeneous nucleation rate is very sensitive to

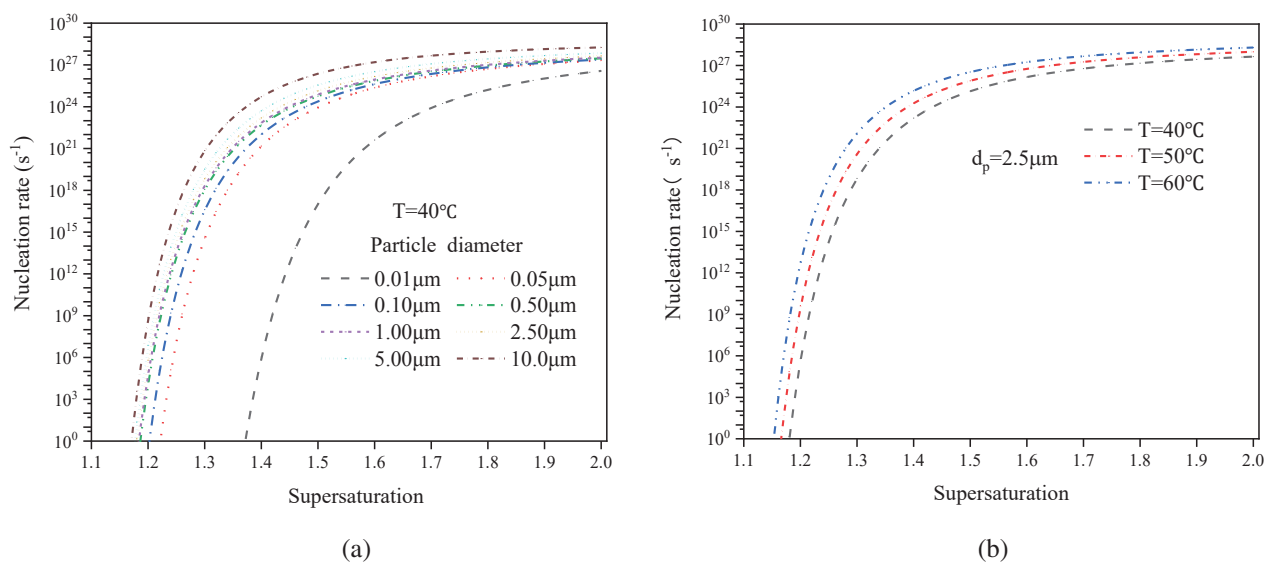


Fig. 6: Variations of nucleation rate with the supersaturation.

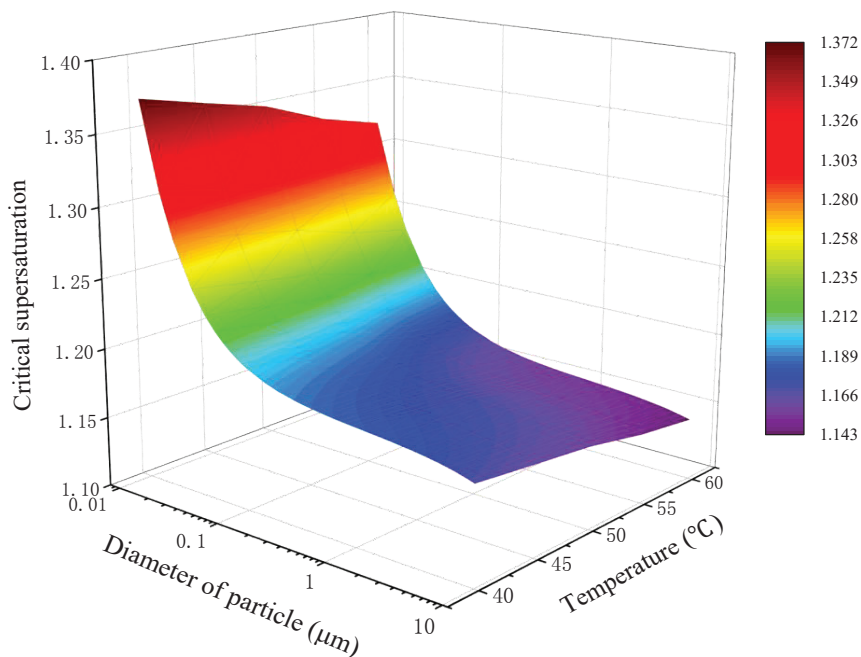


Fig. 7: Variations of critical supersaturation with particle diameter and temperature.

supersaturation, and it increases exponentially with the increase of supersaturation.

4. The critical supersaturation decreases remarkably with the increase of particle diameter when the particle diameter is less than 0.1  $\mu\text{m}$  at different temperatures. With the increase of particle diameter, the trend of the decrease of critical supersaturation tends to level off.
5. The improvement of supersaturated vapour temperature can decrease critical embryo size, increase diffusion condensation rates, thereby increasing the nucleation rate then decreasing the critical supersaturation, which is beneficial to heterogeneous nucleation on coal-fired particles.

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