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Forecasting Precipitation Using a Markov Chain Model in the Coastal Region in Bangladesh

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ABSTRACT

This work explores the detailed study of Bangladeshi precipitation patterns, with a particular emphasis on modeling annual rainfall changes in six coastal cities using Markov chains. To create a robust Markov chain model with four distinct precipitation states and provide insight into the transition probabilities between these states, the study integrates historical rainfall data spanning nearly three decades (1994–2023). The stationary test statistic (χ^2) was computed for a selected number of coastal stations, and transition probabilities between distinct rainfall states were predicted using this historical data. The findings reveal that the observed values of the test statistic, χ^2, are significant for all coastal stations, indicating a reliable model fit. These results underscore the importance of understanding the temporal evolution of precipitation patterns, which is crucial for effective water resource management, agricultural planning, and disaster preparedness in the region. The study highlights the dynamic nature of rainfall patterns and the necessity for adaptive strategies to mitigate the impacts of climate variability. Furthermore, this research emphasizes the interconnectedness of climate studies and the critical need for enhanced data-gathering methods and international collaboration to bridge knowledge gaps regarding climate variability. By referencing a comprehensive range of scholarly works on climate change, extreme rainfall events, and variability in precipitation patterns, the study provides a thorough overview of the current research landscape in this field. In conclusion, this study not only contributes to the understanding of precipitation dynamics in Bangladeshi coastal cities but also offers valuable insights for policymakers and stakeholders involved in climate adaptation and resilience planning. The integration of Markov chain models with extensive historical data sets serves as a powerful tool for predicting future rainfall trends and developing informed strategies to address the challenges posed by changing precipitation patterns.

INTRODUCTION

Bangladesh, located in South Asia, stands as the world's largest deltaic nation, characterized by heavy precipitation owing to its distinctive geographical attributes. The climate is changing both the global (Lambert et al. 2003, Dore 2005) and regional levels (Gemmer et al. 2004) as a result of global warming. In recent years, several research studies have examined precipitation patterns in Bangladesh. The majority of the studies focused on precipitation (Shahid 2011), especially the regional and temporal distribution of monsoon rainfall (Das et al. 2024). The study also examined the fluctuations in yearly rainfall (Shamsuddin & Ahmed 1974), as well as the timing of the entrance and withdrawal of the summer monsoon season (Ahmed & Karmakar 1993), and the variations in rainfall within and between different areas of Bangladesh (Debsarma 2003). Das et al. (2022) carried out research to identify the temporal trends of rainfall in Bangladesh, revealing that the highest rainfall occurs during the monsoon months through nonparametric methodologies. Various probability models have been developed in several studies to depict the distribution of rainfall patterns. From 1989 to 2018, there's been an annual average rainfall decline of 0.014 $mm.y^{-1}$, with increased rainy season rainfall and decreased winter rainfall observed across multiple meteorological stations (Das & Zhang 2021). Most current research relies on analyzing patterns in extreme weather conditions (Khan et al. 2020). For instance, there are differences in the distribution and timing of rainfall along the southwest coast (Hossain et al. 2014) and in other regions of Bangladesh (Sarker & Bigg 2010). The majority of reports were derived from either anecdotal accounts or computer models rather than direct observation. A significant challenge faced by academics is the absence of precise and extensive historical rainfall data from many locations worldwide, which would allow them to differentiate between localized or periodic fluctuations in rainfall trends (Ibeje et al. 2018).

The Markov Chain was used to study the modeling and simulation of various weather phenomena (Gringorten 1996) as well as the development of lengthy time series of weather data (Racsko et al. 1991). The initial stochastic model of temporal precipitation, utilizing a two-state first-order Markov chain, was developed by (Gabriel & Neuman 1962). In 1981, Richardson utilized a first-order Markov chain combined with an exponential distribution to characterize the distribution of daily rainfall in the United States. Akaike (1974) employed a Markov chain model to simulate the daily incidence of rainfall. In addition, the work cited in reference (Sujatmoko & Bambang 2012) employed the methodology of "Statistical Modelling of Daily Rainfall Occurrence". These investigations have demonstrated that by applying the Markov chain combined with an appropriate probability distribution, the produced data accurately maintains the seasonal and statistical properties of historical rainfall data. Several studies have shown that the Markov Chain model is suitable for generating rainfall time series data. A stochastic process is simply a probability process; that is, any process in nature whose evolution we can analyze successfully in terms of probability (Doob 1942). A stochastic process is said to incorporate a Markov chain if it satisfies the characteristics of Markov, often known as the Markovian property. The Markov properties imply that the probability of a future occurrence, given knowledge of past and current events, is independent of previous events and relies on present events (Tovler 2016). The Markov chain is often categorized into two types: The Markov chain with a discrete parameter index and the Markov chain with a continuous parameter index. A Markov chain is considered to have a discrete parameter index when the transition between states happens at specified, discrete time intervals. The Markov chain is said to have a continuous parameter index when the shift state happens within a continuous time interval (Ross 1996). Rainfall data is a temporal dataset that represents the progression of precipitation in a certain region across regular and distinct time intervals.

This research examines a discrete-time four-state model to forecast yearly rainfall patterns and compare them among six coastal cities in Bangladesh. Estimating the probability of rainfall based on current time series data allows us to forecast statistical characteristics such as the mean, standard deviation, and first-order correlation coefficient of rainfall. Accurate assessment of transition probabilities between states at consecutive time occurrences is essential for constructing a model. Theoretical Weibull, Gamma, and Extreme Value Distribution functions are commonly employed in practice and for forecasting rainfall intensity (Villarini et al. 2010). When modeling accounting dependence in a time series, it is common to apply a first-order Markov Chain. Accurate forecasting of future precipitation is necessary to proactively prepare for prolonged periods of high rainfall intensity. In addition, it suggests that we must take into account other factors that might greatly contribute to the escalation of rainfall intensity (Hermawan et al. 2017). A finite Markov chain, a stochastic process with discrete time parameters, was employed in this study to model the yearly rainfall patterns in six coastal cities of Bangladesh. The Markov chain is characterized by the property that the future state of the system depends solely on the present state and is independent of the previous history. The number of states in the process, as defined by Bracken and Croke (2007), can be either limited or countably infinite. The daily precipitation data served as the foundational input for constructing the Markov model, which aimed to simulate the transition of rainfall intensity levels over time. Understanding the intricate variations in rainfall patterns is crucial for multiple sectors in Bangladesh, particularly in coastal areas where the ecology and way of life are significantly influenced by precipitation. Historical rainfall data spanning from 1994 to 2023 were collected for the six coastal cities under investigation: Chittagong, Barishal, Bhola, Cox's Bazar, Khulna, and Patuakhali. The data were meticulously sourced from reputable meteorological databases, governmental archives, and scholarly publications to ensure accuracy and reliability. While previous studies have acknowledged the effectiveness of the Markov model in predicting rainfall, there is a scarcity of research comparing the results of forecasting rainfall using different rainfall states through Markov probability matrices with the outcomes of Markov chain models for future periods. To fill the gaps in past studies, this study establishes the following objectives: (1) To offer further elucidation on modifications in precipitation patterns; (2) To determine the duration required for obtaining limiting state probabilities in rain forecasting; (3) To predict and project rainfall in upcoming periods; (4) To demonstrate the application of the first-order Markov chain model in generating annual rainfall data for future instances. This study proposes an innovative approach for developing prediction models by using various rainfall states derived from the Markov model. The effectiveness of the Markov model in predicting and generating time series data is displayed. The technique employed in this study is transferable to other locations within coastal regions of Bangladesh, as well as to other countries.

MATERIALS AND METHODS

Bangladesh is geographically located in the zone of subtropical climate with the eastern longitude from 88.68°E to 92.97°E and the northern latitude from 20.87°N to 25.78°N (Fig. 1). The country is located in south Asia, which is bordered on the south by the Bay of Bengal, on the southeast by Myanmar, and the remaining by India. Bangladesh is a low plain land comprised of 64 districts. This country is almost entirely flat on a deltaic plain with low elevation and without some hills alongside the Burmese border. This country has a humid subtropical climate; throughout the year, the majority of the country's monsoon weather prevails. As a result, the country's river is, in many instances, flooded with the aid of the tropical cyclones off the Bay of Bengal and with the aid of tidal bores because of its location just south of the foothills of the Himalayas, where monsoon winds turn west, and northwest, the region of Sylhet in eastern Bangladesh receives the greatest average precipitation. From 1994 to 2023, annual rainfall in that region ranged between 3101 and 5944 millimeters per year. The average annual rainfall is 2200 mm. The southwest monsoon is the principal source of rainfall in the districts. About 80% of the total rainfall is received during the period from June to September. From year to year, the variation in the annual rainfall and temperature is not large. In the present study, a series of annual precipitations were analyzed. Most Bangladeshi coastal cities are on riverbanks in low-lying tidal zones at 1.0–1.5 m above sea level. Different coastal regions of Bangladesh house these cities, offering a diversified geographical representation. Including cities from diverse places helps reflect coastal rainfall variability. Rainfall datasets from six weather stations covering the period 1994-2023 were obtained from the Bangladesh Meteorological Department (BMD) in Agargaon, Dhaka. Data is available for the coastal cities of Cox's Bazar, Chittagong, Patuakhali, Bhola, Khulna, and Barishal. The geographical characteristics and locations of all 34 stations in Bangladesh are shown in Fig. 1.

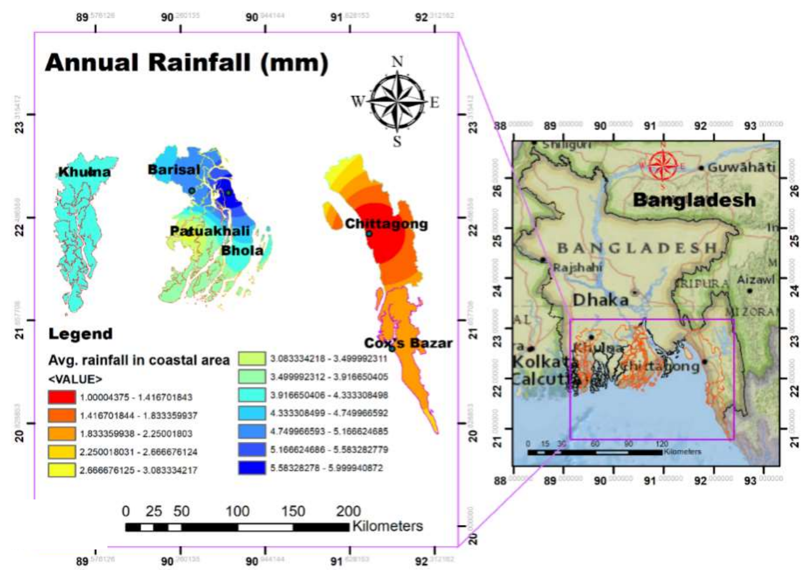


Fig.1: Study region of rainfall stations in Bangladesh.

Table 1: Statistics of the annual rainfall data for the six stations.

Stations	Descriptive Measure					
	Maximum	Minimum	Mean	CV[%]	Skewness	Kurtosis
Chittagong	3833	2208	2953	16.79	0.178	-1.102
Barisal	2858	1418	2057.581	18.23	0.156	-0.842
Bhola	3080	1493	2156.453	17.704	0.296	-0.441
Cox's Bazar	4716	2483	3728.903	14.178	-0.268	-0.159
Khulna	2594	1073	1806.226	19.578	-0.188	-0.196
Patuakhali	3098	1847	2547.419	13.962	-0.499	-0.823

Table 1 provides an analysis of annual rainfall data from six meteorological stations: Chittagong, Barishal, Bhola, Cox's Bazar, Khulna, and Patuakhali. Statistical summaries include maximum, minimum, and mean annual rainfall levels, coefficient of variation (CV %), skewness, and kurtosis for each station. For instance, Chittagong's maximum annual rainfall is 3833mm, with a mean of 2953mm and a coefficient of variation of 16.79%. Barishal has a maximum of 2858mm, a mean of 2057.581mm, and a slightly higher coefficient of variation at 18.23%. These insights offer valuable data for meteorological and climate research.

Methodology

Markov Chain Modeling

The study's methodology is rooted in the theoretical framework of Markov chains, with a focus on transition probabilities, steady-state probabilities, and limiting-state probabilities. The analysis relies on the following definitions, theorems, and equations:

A Markov chain is characterized as a random sequence $(X_n, n \in N)$ where each state X_n is dependent solely on the preceding state X_{n-1}. This Markov property asserts that the future state is conditionally independent of past states, given the present state.

Transition probabilities: In a consistent Markov chain $(J_n, n \geq 0)$, transition probabilities from state i to state j are denoted as P_{ij}. The transition matrix $P = [P_{ij}]$ encapsulates all transition probabilities between states i and j.

Steady-state transition probabilities: Steady-state transition probabilities are observed in the Markov process X when the n-step transition probability. P_{ij}^n satisfies the condition $P_{ij}^n = P\{X_{n+m} = J / X_m = i\}$ for all $n, m \geq 0$, and all states $i, j \geq 0$.

$$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^n P_{kj}^m = 0 \ for\ all\ n, m \geq 0 \quad ...(1)$$

for all $n, m \geq 0$, establishes the relationship between n -step and m-step transition probabilities in a Markov chain.

Induction and limiting state probability: Through induction, it is shown that $P^n = P^0 P^n$, where P^0 represents the initial state vector of the transition matrix. The limiting state probability is denoted as $P^n = [P_1^n \quad P_2^n \quad P_3^n \quad P_4^n]$, indicating the probabilities of reaching each state after n steps.

A dataset was compiled for each city to define four rainfall states in a Markov chain model. These states were categorized as follows: State 1 represents low rainfall, State 2 denotes moderate rainfall that is evenly distributed, State 3 indicates heavy rainfall, and State 4 represents moderate rainfall that is not evenly distributed. Transition probabilities were depicted in a diagram and matrix, strategically incorporating zeros to represent no direct transitions between certain states. Regional climate variability, ecological impacts, historical data, sector-specific factors, and modeling objectives influenced the state definitions. This methodology aimed to capture nuanced rainfall variations and their implications for coastal cities. The transition diagram in Fig. 2 and the probability matrix P depict the transition between states.

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & 0 \\ P_{41} & P_{42} & 0 & 0 \end{bmatrix}$$

Zeros in the probability matrix denote impossible transitions between certain rainfall states. For example, State 4 doesn't transition directly to State 3 or State 1, reflecting the constraints of realistic rainfall patterns observed in coastal cities. These zeros shape the Markov model, ensuring meaningful state transitions based on observed rainfall characteristics.

RESULTS AND DISCUSSION

In this study, we analyze the patterns of rainfall distribution in key coastal cities of Bangladesh, namely Chittagong, Barishal, Bhola, Cox's Bazar, Khulna, and Patuakhali (Table 2). Utilizing data on rainfall measurements and frequency of occurrences, the study unveils distinctive precipitation trends across these urban centers. Findings indicate varying ranges of rainfall, with Chittagong experiencing a broad spectrum of precipitation, while Cox's Bazar demonstrates a more concentrated pattern. Barishal and Bhola exhibit similar rainfall tendencies, with notable peaks in specific ranges. Khulna showcases a diversified rainfall regime, reflecting its adaptive capacity, while Patuakhali witnesses substantial precipitation occurrences. These insights underscore the importance of tailored urban planning and disaster preparedness strategies to address climatic vulnerabilities in Bangladesh's coastal regions.

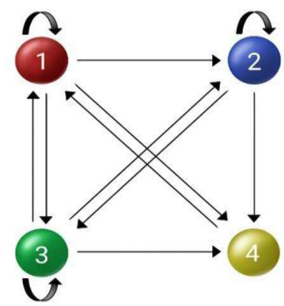


Fig. 2: Transition Diagram.

Limiting State Probabilities

The Markov chain model provides a succinct and probabilistic framework for comprehending and perhaps forecasting rainfall patterns. To simulate more intricate rainfall dynamics and align with specific research goals, the model may be adjusted by integrating more states or altering transition probabilities.

Chittagong: The assumed model for annual rainfall in the Chittagong area is State-1: (2200-2745) mm**,** State-2: (2746-3290) mm, State-3: (3291-3845) mm, State-4: (2746-3290) mm. Therefore, the transition matrix

$$M = \begin{bmatrix} 5 & 3 & 2 & 3 \\ 0 & 1 & 5 & 0 \\ 6 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

i.e., $P_{ij} = \frac{f_{ij}}{f_{ij}\,\Sigma f_{ij}}\; i,j = 1,2,3,4.$ (Arumugam & Karthik 2018) Where $f_{ij} \rightarrow$ transition frequency from state to state j, $0 \leq P_{ij} \leq 1$. We get the probability matrix

$$P = \begin{bmatrix} 0.38 & 0.23 & 0.16 & 0.23 \\ 0 & 0.16 & 0.84 & 0 \\ 0.75 & 0.125 & 0.125 & 0 \\ 0.67 & 0.33 & 0 & 0 \end{bmatrix} \quad ...(2)$$

N-step transition probability, we have

$$P^2 = \begin{bmatrix} 0.419 & 0.220 & 0.274 & 0.087 \\ 0.63 & 0.131 & 0.239 & 0 \\ 0.378 & 0.208 & 0.241 & 0.173 \\ 0.255 & 0.207 & 0.384 & 0.154 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.439 & 0.196 & 0.267 & 0.097 \\ 0.437 & 0.206 & 0.261 & 0.964 \\ 0.425 & 0.196 & 0.277 & 0.1 \\ 0.422 & 0.195 & 0.271 & 0.112 \end{bmatrix}$$

$$P^6 = \begin{bmatrix} 0.433 & 0.198 & 0.269 & 0.099 \\ 0.434 & 0.197 & 0.268 & 0.098 \\ 0.432 & 0.198 & 0.269 & 0.100 \\ 0.431 & 0.198 & 0.270 & 0.101 \end{bmatrix}$$

And

$$p^8 = \begin{bmatrix} 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \end{bmatrix}$$

In a Markov chain method, here, two successive iterations yield identical results, which signifies convergence toward the limiting state. This indicates stable probabilities of transitioning between states, suggesting further iterations are unlikely to alter the state distribution significantly. The system has reached a consistent state, implying its long-term behavior has been established, regardless of additional iterations**.** After *n* steps, P^0 gets the fixed value (3), i.e., n ≥ 6

Let us take $P^0 = (1\ \ 0\ \ 0\ \ 0)$

$$P^0 P^n = (1 \quad 0 \quad 0 \quad 0) \begin{bmatrix} 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \\ 0.433 & 0.198 & 0.269 & 0.1 \end{bmatrix}$$

$$= (0.433 \quad 0.198 \quad 0.269 \quad 0.1)$$

Thus, $n = nP$= (0.433 0.198 0.269 0.1) These results show the yearly rainfall probability after 6 years. In the first year, the likelihood is (0.433 0.198 0.269 0.1). When the odds were compared, state 4 declined steadily while the likelihood of states 1 and 2 increased, exceeding six years. This means that 43% of the yearly rainfall in Chittagong will fall on State 1, 20% on State 2, 27% on State 3, and 10% on State 4. Similarly, we have five coastal cities: Barishal, Bhola, Cox's Bazar, Khulna, and Patuakhali, and their probability matrices are P_B, P_{Bh}, P_{CB}, P_K, P_P.

Barishal: Let the model for yearly rainfall for the Barishal region be State-1: (1415-1900) mm, State-2: (1901-2385) mm, State-3: (2386-2870) mm, State-4: (1901-2385) mm. Therefore, the transition matrix

$$M = \begin{bmatrix} 4 & 5 & 4 & 3 \\ 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$$

Table 2: Frequency of annual rainfall in six coastal cities between 1994-2023.

State	Chittagong		Barishal		Bhola		Cox's Bazar		Khulna		Patuakhali	
	Rainfall [mm]	Frequency	Rainfall [mm]	Frequency	Rainfall [mm]	Frequency	Rainfall [mm]	Frequency	Rainfall [mm]	Frequency	Rainfall [mm]	Frequency
1	2200-2745	13	1415-1900	13	1490-2025	14	2480-3230	5	1130-1620	10	2100-2435	10
2	2746-3290	6	1901-2385	5	2026-2560	6	3231-3980	11	1621-2111	10	2436-2770	8
3	3291-3845	8	2386-2870	6	2561-3095	4	3981-4730	10	2112-2602	3	2771-3105	9
4	2746-3290	3	1901-2385	6	2026-2560	6	3231-3980	4	1621-2111	7	2436-2770	3

And get the probability matrix.

$$P_B = \begin{bmatrix} 0.25 & 0.313 & 0.25 & 0.188 \\ 0 & 0.2 & 0.4 & 0.4 \\ 0.5 & 0.34 & 0 & 0.16 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \ldots (3)$$

After n steps, P^0 gets the fixed value i.e., n ≥ 4 also gets (0.39 0.23 0.19 0.19) four stages are limiting stage probabilities. The resultant values show the 4 years annual probability distribution of rainfall. The odds for each state in the first year are spread as (0.39 0.23 0.19 0.19). A comparison study shows that the chance of State 4 has consistently decreased over the decade, while the probability of States 1 and 2 has increased. As a result, after ten years, 39% of yearly rainfall in Barishal is expected to fall in State 1, 23% in State 2, 19% in State 3, and 19% in State 4.

Bhola: Let the model for yearly rainfall for the Bhola region be State-1: (1490-2025) mm, State-2: (2026-2560) mm, State-3: (2561-3095) mm, State-4: (2026-2560) mm. Consequently, the transition matrix

$$M = \begin{bmatrix} 7 & 1 & 3 & 3 \\ 0 & 2 & 1 & 2 \\ 1 & 3 & 0 & 0 \\ 6 & 0 & 0 & 0 \end{bmatrix}$$

And probability matrix

$$P_{Bh} = \begin{bmatrix} 0.5 & 0.08 & 0.21 & 0.21 \\ 0 & 0.4 & 0.2 & 0.4 \\ 0.25 & 0.75 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \ldots (4)$$

After n steps, P^0 receives the fixed value i.e., n ≥ 4 also obtains the four-stage limiting stage probability The expression (0.44 0.23 0.14 0.19) is the solution to equation (4), which represents the yearly probability distribution of rainfall after four years. The probability in the first year is (0.44 0.23 0.14 0.19). When the probability is compared, State 4 shows a continuous reduction, but States 1 and 2 show an increase during the four years. As a result, the Bhola projection suggests that 44% of the yearly rainfall will fall in State 1, 23% in State 2, 14% in State 3, and 19% in State 4.

Cox's Bazar: In the Cox's Bazar region, the yearly rainfall model is State-1: (2480-3230) mm, State-2: (3231-3980) mm, State-3: (3981-4730) mm, State-4: (3231-3980) mm. Consequently, the transition matrix

$$M = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 6 & 4 & 1 \\ 1 & 4 & 5 & 0 \\ 3 & 1 & 0 & 0 \end{bmatrix}$$

and probability matrix

$$P_{CB} = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.4 \\ 0 & 0.54 & 0.36 & 0.1 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0.75 & 0.25 & 0 & 0 \end{bmatrix} \quad \ldots (5)$$

After the n steps, P^0 receives the fixed value, i.e., n ≥ 8, also obtains the four-stage limiting stage probability (0.13 0.42 0.35 0.1). The probability distribution (0.1334 0.4175 0.354 0.0951) for Cox's Bazar over eight years is obtained by solving equation (5). An eight-year comparison demonstrates a consistent reduction in State 4, with a rise in odds for States 2 and 3. As a result, Cox's Bazar expects 13% of its annual rainfall to fall in State 1, 42% in State 2, 45% in State 3, and 10% in State 4.

Khulna: The annual rainfall model for the Khulna area is State-1: (2200-2745) mm, State-2: (2746-3290) mm, State-3: (3291-3845) mm, State-4: (2746-3290) mm. Thus, the transition matrix

$$M = \begin{bmatrix} 1 & 7 & 1 & 5 \\ 0 & 7 & 1 & 2 \\ 2 & 0 & 1 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$$

and probability matrix

$$P_K = \begin{bmatrix} 0.1 & 0.3 & 0.1 & 0.5 \\ 0 & 0.7 & 0.1 & 0.2 \\ 0.67 & 0 & 0.33 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \ldots (6)$$

The fixed value of (6) is obtained after n steps, and the four-stage limiting stage probabilities (0.33 0.33 0.11 0.23) are obtained as well when n $\geq$ 12. The solution of equation (6) can be expressed as the expression. (0.33 0.33 0.11 0.23), reflecting Khulna's yearly rainfall probabilities over twelve years. Probabilities correspond with this solution in the twelfth year. A comparison study reveals that State 4 has been steadily declining throughout the decade, while States 1 and 2 have maintained equal odds. As a result, Khulna predicts that 33% of yearly rainfall will fall in State 1, 33% in State 2, 11% in State 3, and 23% in State.

Patuakhali: For the Patuakhali region, the yearly rainfall model is State-1: (2200-2745) mm, State-2: (2746-3290) mm, State-3:(3291-3845) mm, State-4: (2746-3290) mm. Thus, the transition matrix

$$M = \begin{bmatrix} 4 & 1 & 4 & 1 \\ 0 & 3 & 4 & 1 \\ 3 & 4 & 1 & 1 \\ 3 & 0 & 0 & 0 \end{bmatrix}$$

and probability matrix

$$P_P = \begin{bmatrix} 0.4 & 0.1 & 0.4 & 0.1 \\ 0 & 0.375 & 0.5 & 0.125 \\ 0.34 & 0.44 & 0.11 & 0.11 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad ...(7)$$

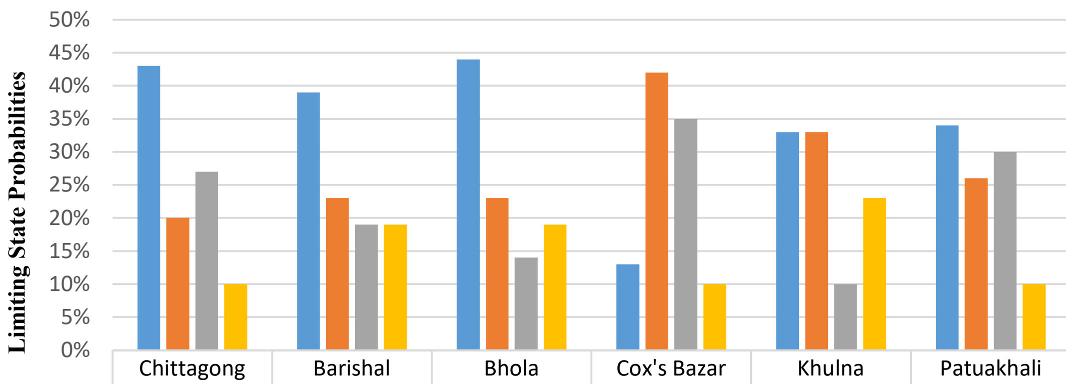
When n is greater than or equal to 4, not only is the fixed value (7) reached after n steps, but the four-stage limiting stage probability. (0.34 0.26 0.3 0.1) is also acquired. Finally, solving equation (7) produces the probability distribution. (0.34 0.26 0.3 0.1) For Patuakhali s yearly rainfall over four years. A comparison analysis shows a consistent reduction in State 4, whereas States 1 and 3 see an increase in probability during the ten years. As a result, Patuakhali predicts that 34% of yearly rainfall will fall in State 1, 26% in State 2, 30% in State 3, and 10% in State 4.

Fig. 3 illustrates the probability distribution of each state (state-1, state-2, state-3, and state-4) among six distinct study areas. The likelihood of each state is depicted by distinct bars. By comparing the heights of these bars for each district, we may gain insight into the probability distribution of states in each coastal city. The graph illustrates four discrete situations of rainfall limitation, which may be classified according to the magnitude of rainfall. Let us designate them as State 1, State 2, State 3, and State 4. The graph represents six districts: Chittagong, Barishal, Bhola, Cox's Bazar, Khulna, and Patuakhali. These districts are probably coastal areas, as previously noted. The figures depicted in the graph denote the odds of transitioning between states within each district. Each row corresponds to a certain district, while each column represents an individual state. For instance, the number in the initial row and second column (State 2) represents the chance of moving from State 1 to State 2 in the Chittagong area. The percentages displayed in each cell of the graph indicate the probability of shifting from one state to another within a certain district. For example, examining the number in the third row and fourth column (State 4) reveals the chance of moving from State 3 to State 4, specifically in the Bhola area. The Markov chain model is applicable for the analysis and prediction of rainfall patterns in specified areas. By analyzing the transition probabilities, it is possible to determine the probability of distinct rainfall conditions happening in each district. This information holds significant value for many applications, including agricultural planning, water resource management, infrastructure building, and disaster preparedness. To summarize, the above graph depicts a Markov chain model that showcases the odds of transitioning between different states of rainfall limitation across six districts. It offers valuable information on the probability of distinct rainfall conditions happening in each district, assisting in decision-making processes for sectors affected by rainfall patterns.

In summary, this section elucidates the foundational principles of Markov chains, emphasizing the importance of transition probabilities, steady-state probabilities, and limiting-state probabilities in analyzing the behavior and stability of dynamic systems. These theoretical constructs serve as the cornerstone for our subsequent application of Markov models in investigating complex processes and phenomena.

Stationary Test for Rainfall Occurrences

Since satisfactory crop yields mainly depend on the pattern of rainfall occurrences, it is necessary to know whether the



	Chittagong	Barishal	Bhola	Cox's Bazar	Khulna	Patuakhali
state-1	43%	39%	44%	13%	33%	34%
state-2	20%	23%	23%	42%	33%	26%
state-3	27%	19%	14%	35%	10%	30%
state-4	10%	19%	19%	10%	23%	10%

Fig. 3: Observed state probabilities for six coastal regions of Bangladesh.

rainfall occurrences are stationary or not to estimate the crop yields for later periods. Thus the stationary test has been employed on rainfall data to test the null hypothesis that rainfall occurrences are stationary against the alternative that rainfall occurrences are not stationary.

$$\text{i.e., } H_0: P_{ij}(t) = P_{ij}$$

$$H_1: P_{ij}(t) \neq P_{ij}$$

For all selected stations, let us consider the transition probabilities. $\widehat{P_{ij}}$ and $\widehat{P_{ij}}(t)$ which are estimated from the rainfall data for 30 years (1994-2023). Using these probabilities, the values of stationary test statistic χ^2 have been calculated for the selected stations, which are shown in Table 3. The χ^2 values are found to be insignificant for coastal stations. Thus we may conclude that rainfall occurrences are stationary for all the stations.

Table 3: Values of Stationary Test Statistic, χ2 for Selected Stations.

Stations	Observed values of χ^2	Degrees of Freedom	P value
Chittagong	18.66**	9	0.028
Barishal	16.91	9	0.050
Bhola	20.76**	9	0.014
Cox's Bazar	19.98**	9	0.018
Khulna	31.31**	9	0.00026
Patuakhali	21.65**	9	0.01
** Insignificant at 0.05 level.			

The stationary test was conducted to examine the null hypothesis () that the rainfall occurrences are stationary against the alternative hypothesis () that the rainfall occurrences are not stationary. This test is crucial for estimating crop yields in later periods, as the pattern of rainfall occurrences is a key factor influencing agricultural productivity. The results show that the observed values of the test statistic, χ^2, are found to be significant for all coastal stations except Barishal at a 5% level of significance with 7 degrees of freedom.

CONCLUSIONS

To sum up, the investigation of Bangladesh's precipitation patterns using finite Markov chains has provided insight into the complicated mechanisms of the region's rainfall variability. Through the use of advanced modeling tools and historical data, this work has yielded useful data about the temporal history of rainfall patterns, providing a detailed understanding of the fluctuations in precipitation that occur in various states and regions. The statistical studies carried out in this study highlight the complicated nature of Bangladesh's rainfall patterns by identifying lighter-tailed distributions and leftward skewness in the data. The results of the stationary show that the observed values of the test statistic, χ^2, are found to be significant for all coastal stations except Barishal. We conclude that the rainfall occurrences are stationary for Barishal stations and other stations; the rainfall occurrences are non-stationary.

By filling data gaps and fostering international collaborations, scientists can enhance predictive models, shedding light on how climate change will impact rainfall patterns. This study enhances understanding of Bangladeshi precipitation dynamics, underscoring the importance of data-driven insights and advanced modeling in navigating climate complexity. With ongoing innovation and collaboration, we can pursue sustainable solutions, fostering a more adaptable and stable society amidst environmental shifts.

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