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Development of Crown Profile Models for Chinese Fir Using Non-linear Mixed-Effects Modelling

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ABSTRACT

Crown profile models are key components of growth and yield models and are crucial for estimating the crown volume and constructing 3D visualization of trees. We used a total of 431 trees collected from 98 pure even-aged temporary sample plots established in Fujian Province to develop crown profile models of Chinese fir (*Cunninghamia lanceolata*).To describe the shape of tree crowns more accurately, significance tests of the effects of different stand conditions (stand age, site index, and stand density) on crown shape were conducted with one-way analysis of variance (ANOVA). Multiple comparisons based on the ANOVA results were used to classify the crown data into three groups according to stand age: Group I (young forest), Group II (medium forest), and Group III (nearly mature and mature forest). We analysed the relationships between the crown variables and stand variables and used the reparameterization approach to develop three optimal crown profile models for different age groups. Stand variables (such as stand density) further improved the prediction efficacy of the models. Considering the correlation between repeated measurement data for the same tree crown, the non-linear mixed-effects modelling (NLME) method was used to account for autocorrelation. The determination coefficients (R^2) of the above three optimal models fitted by the non-linear mixed-effects approach were 0.9214, 0.9398 and 0.9129, and their Root Mean Squared Errors (RMSEs) were 0.1246, 0.1409 and 0.1786, respectively. The determinant coefficients (R^2) of the three models fitted by the non-linear least squares (NLS) approach were 0.9015, 0.8794 and 0.8930, and their RMSEs were 0.1395, 0.2102 and 0.1878, respectively. The results indicated that the predicted accuracy was significantly increased by using non-linear mixed effects modelling compared with the NLS method.

INTRODUCTION

Tree crowns play an essential role in the photosynthesis and transpiration processes, and crown structure can not only intuitively reflect the vigour of trees (Biging 1995), but is also an indicator of inter-tree competition level (Purves 2007). The size and shape of a tree's crown are used in ecological research to simulate the above-ground biomass of trees (Carvalho 2003) and the interception of solar radiant energy (Pretzsch 2014). In addition to these applications, crown modelling is also crucial for estimating crown volume and constructing 3D visualization of trees in forest management (Dong 2016).

Crown profile models depict the shape and size of trees by predicting the radius of the crown at any height along the entire crown length (Gao 2017). These models generally fall into simple geometric models, segmented models and variable exponential models according to the equation forms. Earlier attempts that adopted simple regular geometrical shapes described the change in crown radius from the top to the base of the crown. Mohren (1987) and Marshall (2003) suggested the following model to estimate crown shape: $CR_i = (1 - di)^k$ (1-1), where d_i represents the relative crown length, and k is the model coefficient. The value of k characterized the shape of crowns for a range of geometric solids (i.e., cylinder, parabola, cone, neiloid). Nepal (1993) proposed that the location of the largest crown radius was not at the base of the tree crown and extended Equation 1-1 for the Loblolly pine (*Pinus taeda* L.). To describe the crown structure more flexibly, a variety of equations have been used such as polynomials (Baldwin 1997, Dong 2016), power functions (Rautiainen 2005), peak functions (Guo 2015) and modified Beta functions (Ferrarese 2015). Simple geometric models were thought to be excessively rigid, and researchers (Raulier 1996, Pretzsch 2002, Crecente-Campo 2013, Sadono 2015a, 2015b) adopted a segmented approach by directly dividing the entire crown into upper (i.e., mostly sun needles) and lower (i.e., mostly shade needles) crowns at the location of the largest crown radius. Gao et al. (2015) concluded that the artificially selected point in a segmented model is not differentiable, and continuous segmented models were proposed to solve this problem. However, the model fitting of segmented equations is complex. Ferrarese (2015) and Gao (2018) used variable exponent models to represent crown shapes by changing the parameters. Compared with segmented models, variable exponent models are simpler in form, although the equation is not integral when estimating the crown volume.

Researchers have developed crown profile models for many tree species. Chinese fir is an important timber tree species in southeast China. With few exceptions, these models for Chinese fir have been estimated by ordinary least squares (OLS) or non-linear least squares (NLS). Guo (2015) and Dong (2016) used the NLS framework to develop crown profile models for Chinese fir. The relationship between crown radius and independent variables (stand variables and tree variables) is usually determined by measurements from the same tree crown. The homologous structure (i.e., measurements within a tree crown) destroys the independence between observations and invalidates the basic hypothesis of the least squares methods (West et al. 1984). Therefore, OLS and NLS tend to inflate the standard errors in the estimated parameters (Calama & Montero 2004). Non-linear mixed-effects (NLME) modelling was proposed to solve this problem (Lindstrom & Bates 1990). NLME modelling contains both fixed and random effects and provides an efficient approach to analyse repeated measurements from the same statistical unit. However, to our knowledge, few studies have applied NLME modelling to develop a crown profile model for Chinese fir.

The objective of this study was to develop a crown profile model for even-aged Chinese fir in Fujian Province (southeast China) and use NLME modelling to enhance the existing crown profile models. To describe the shape of tree crowns more accurately, we used ANOVA and multiple comparison methods to classify the crown data. Many candidate equations were compared, and the equations that performed best were selected for further analysis. The reparameterization approach was used to develop the basic crown profile models of the NLME modelling based on the models selected. We further analysed the 2D and 3D crown shapes under different forest stand growth conditions.

MATERIALS AND METHODS

Study Area and Data Collection

The study area is located in the Dali forest farm and Lanxia forest farm (115°50'~120°40'E, 23°33'~28°20'N) in Fujian province, Southeast China. The main terrain consists of hills and fewer plains with elevations between 300m and 500m. With a subtropical maritime monsoon climate, the annual average temperature of this area is 20°C and the annual average precipitation is 1756 mm. Soil types mainly include red soil, yellow soil and mountain meadow soil. The main wood species are Chinese fir, Masson's pine and *Eucalyptus grandis*.

Data for the current study were collected from 98 temporary sample plots that were established according to different age groups, stand densities and site condition types in 2015. The size of each plot was 20m×30m. Then, 4~5 trees without deflection crowns were selected from each plot for crown modelling, and a total of 413 trees were collected. For each tree, the following measurements were obtained: (1) diameter at breast height (*DBH*, cm), defined as 1.3m; (2) total tree height (*HT*, m); (3) largest crown radius (*LCR*, m) which was half of the crown width (*CW*, m); (4) height above ground to the crown base (*HCB*, m); (5) crown length (*CL*, m); (6) vertical height from the crown base to the crown position at p**CL* (*CH*, m), where p is equal to 1/10, 1/4, 2/4, 3/4 and 9/10; and (7) crown radius, corresponding to *CH* (*CR*, m). *DBH* was measured to the nearest 0.1 cm and other variables were measured to the nearest 0.1m. The data were randomly divided into the model development dataset and the testing dataset with a ratio of 8:2. The crown variables are shown in Fig. 1 and stand attributes of the 98 sample plots are described in Table 1.

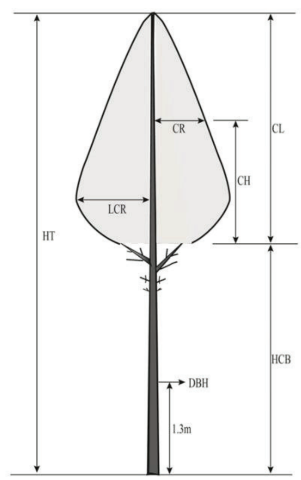


Fig. 1: Variables used to characterize an individual tree crown.

Table 1: Summary statistics of measurements of tree variables.

Tree variables	Mean	Min	Max	STD
t (years)	16	5	29	7.25
N(treesha^{-1})	1850	900	4000	60.39
DBH (cm)	16.4	5.9	33.2	5.52
HT(m)	12.6	3.0	25.5	3.91

Table cont....

CW(m)	3.6	1.6	7.6	1.05
LCR(m)	1.8	0.8	3.8	0.53
CL(m)	6.0	1.1	14.3	2.65
HCB(m)	6.4	0.3	16.3	3.17
CR(m)	1.1	0.1	3.8	0.56

Note: t = tree age, N = stand density, DBH = diameter at breast height, HT = total tree height, CW = crown width, LCR = largest crown radius (1/2CW), CL = crown length, HCB = height above ground to crown base, CR = crown radius, Min = minimum, Max = maximum, Mean = average value, and STD = standard deviation.

METHODS

ANOVA and Multiple Comparisons

Tree crown shape and size vary with forest stand density, management measures and tree ages. To accurately simulate the crown profile, significant tests of different stand conditions on crown profiles were conducted with one-way ANOVA at a significance level of 0.05, and LSD multiple comparisons were used to further group the crown data.

The stand site index (*SI*), the initial stand density (*SD*) and the tree age were selected as factors for ANOVA. The *LCR* and *CL* from sample data were taken as the observation variables. The *SI* was divided into 7 levels: 10(9<*SI*<=11), 12(11<*SI*<=13), 14(13<*SI*<=15), 16(15<*SI*<=17), 18(17<*SI*<=19), 20(19<*SI*<=21) and 22(21<*SI*<=22). According to the criteria for classification of forest age groups, the tree ages of Chinese fir were divided into 4 levels: young forest (<=10), medium forest (11-20), nearly mature forest (21-25) and mature forest (26-35) years. The *SD* included 6 levels: I (<1000), II (1001-1500), III (1501-2000), IV (2001-2500), V (2501-3000) and VI (>3000).

In this paper, single factor ANOVA was used to study whether there were significant differences in crown radii under different growth conditions, taking crown radius analysis under different age groups as an example. The statistical equations of ANOVA and LSD multiple comparisons are shown in Equation 1 and 2:

$$F = \frac{MSA}{MSE} \sim F\left(k-1, n-k\right) \qquad \ldots(1)$$

Where, MSA is the mean square between groups, MSE is the intra-group mean square, n is the number of observed values, and k is the number of groups.

$$LSD = t_{\alpha/2}\sqrt{MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)} \qquad \ldots(2)$$

Where, $t_{a/2}$ is the critical value of the t distribution, MSE is the mean square residual in the group, and n_i and n_j represent the sizes of the ith and jth samples, respectively.

Development of the Optimal Crown Profile Models

In this study, 24 candidate model equations (Table 2) were collected, and the basic crown profile models of three modelling forms were analysed: (i) simple single equations (models 1-12), (ii) segmented equations (models 13-19), and (iii) variable-exponent equations (models 20-24). To choose the optimal models that performed best for different age groups, we compared the candidate models from Table 2 using the different age group data according to the principle of R^2 maximization and RMSE minimization. In addition, stand variables were assessed with respect to their influence on crown profiles. Therefore, we used the reparameterization method to analyse the relationships between the optimal model parameters

Table 2: Crown profile candidate models.

No.	Model equations	References
1	$CR = LCR\left(1-RCH\right)^{a_0}$	Mohren (1987)
2	$CR = LCR\left(1-RCH^2\right)^{a_0}$	McPherson (1988)
3	$CR = a_0 RCH^{a_1}\left(1-RCH\right)^{a_2}$	Nepal (1993)
4	$CR = a_0\left(RCH-1\right)/\left(RCH+1\right)+a_1\left(RCH-1\right)$	Baldwin (1997)
5	$CR = LCR(a_0\left(RCH-1\right)/\left(RCH+1\right)+a_1\left(RCH-1\right))$	Crecente-Campo (2009)
6	$CR = LCR\left(a_0\left(RCH-1\right)/\left(RCH+1\right)+a_1\left(1-RCH\right)^{a_2}\right)$	Chmura (2014)
7-1	$CR = LCR\left(a_0+a_1 RCH+a_2 RCH^2\right)$	Crecente-Campo (2009)

Table cont....

No.	Model equations	References
7-2	$CR = LCR\left(a_0 + a_1RCH + a_2RCH^2 + a_3RCH^3\right)$	Crecente-Campo (2009)
7-3	$CR = LCR\left(a_0 + a_1RCH + a_2RCH^2 + a_3RCH^3 + a_4RCH^4\right)$	Crecente-Campo (2009)
8	$CR = LCR\left(a_0\left(RCH-1\right) + a_1\left(RCH^2-1\right) + a_2\left(RCH^3-1\right) + a_3\left(RCH^4-1\right)\right)$	Crecente-Campo (2009)
9	$CR = LCR\left(a_0 - a_1\ln\left(RCH + a_2\right)\right)$	Guo (2015)
10	$CR = LCR\left(a_0 + a_1RCH^{a_2}\right)$	Guo (2015)
11	$CR = a_0 - \frac{a_1}{\left(1 + a_2RCH\right)^{1/a_3}}$	Rautiainen (2005) Guo (2015)
12	$CR = a_0\frac{\left(1-RCH\right)^{p-1}RCH^{q-1}}{\beta\left(p,q\right)}$	Ferrarese et al. (2015)
13	$CR_U = \mathrm{LCR}\left(1 - \left(\frac{CH - L_L}{L_U}\right)^{b_0}\right)^{1/b}$	Rautiainen (2005)
14	$CR_U = LCR\left(\frac{\left(LCL - CH\right)}{L_U}\right)^{b_0}$	Crecente-Campo (2013)
15	$CR_L = LCR\left(b_0 + \left(1-b_0\right)\left(\frac{CH}{L_L}\right)^{b_1}\right)$	Crecente-Campo (2009)
16	$CR_L = LCR\left(CH^{b_0} + b_1\right)$	Dong (2016)
17	$CR = b_0\left(1-RCH\right) + b_1\left(1-RCH\right)^2 + b_2\left(1-k-RCH\right)^2 I_+$	Gao (2015)
18	$CR = b_0\left(1-e^{-b_1(1-RCH)}\right) - b_2\left(1-e^{-b_3(1-k-RCH)}\right)I_+$	Gao (2015)
19	$CR = b_0\left(1-RCH\right)^{b_1} - b_2\left(1-k-RCH\right)^{b_3} I_+$	Gao (2015)
20	$CR = LCR\left(e^{c_0 + c_1RCH + c_2RCH^2}\right)$	Hann (1998)
21	$CR = LCR\left(c_0 + c_1e^{c_2RCH}\right)$	Guo (2015)
22	$CR = c_0 / \left(1 + e^{-c_1(RCH - c_2)}\right)$	Guo (2015)
23	$CR = c_0\left(\frac{1-RCH^{0.5}}{1-c2^{0.5}}\right)^{(c_1RCH)}$	Gao (2015)
24	$CR = c_0\left(\frac{c_2}{c_1}\right)\left(\frac{1-RCH}{c_1}\right)^{c_2-1} e^{-\left(\frac{1-RCH}{c_1}\right)^{c2}}$	Ferrarese et al. (2015)

Note: CR_U and CR_L are the crown radii of the upper and lower parts, m; *Lu* is the crown length of the upper half of the tree, m; L_L is the crown length of the lower half, m; *RCH* is relative crown length (*RCH=CH/LCL*, 0 at the base of crown and 1 at the top of crown), m;a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, c_0, c_1, c_2, c_3 are the model coefficients; in models 17,18,19, $I_+ = \begin{cases} 0 & RCH < k \\ 1 & RCH \geq k \end{cases}$, *k* is the inflection point parameter of the segmented model, i.e., the maximum crown position point; in the model 12 (*p*, *q*) is a standardized function of beta distribution with parameters *p*> 0 and *q*> 0; *e* is the natural logarithm, 2.718.

and stand variables.

The entire analysis procedure was done in three steps. First, to obtain the estimated values of model parameters corresponding to each tree, the interpolation method was used to obtain the data between the measurement points at different positions of the crown (Gill & Biging 2002). After comparison with the commonly used interpolation algorithms, such as linear interpolation, Hermite interpolation and cubic spline interpolation, the Lagrange interpolation was selected. The optimal model was fitted with the interpolated data, and the corresponding model parameters of each tree were obtained. Assuming that a given k+1 measuring points at different crown positions, (x_0, y_0), …, (x_k, y_k), are known. The Lagrange interpolation function is shown in Equations 3 and 4:

$$L(x)=\sum_{j=0}^{k} y_j l_j(x) \quad \ldots(3)$$

Where, $l_j(x)$ is an interpolation basis function and

$$l_j(x)=\prod_{i=0,i\neq j}^{k}\frac{x-x_i}{x_j-x_i}=\frac{(x-x_0)}{(x_j-x_0)}\cdots\frac{(x-x_{j-1})}{x_j-x_{j-1}}\frac{(x-x_{j+1})}{x_j-x_{j+1}}\cdots\frac{(x-x_k)}{x_j-x_k} \quad \ldots(4)$$

Where, x_j is the jth observation of crown length measurement for each tree and y_j is the jth observation of crown radius measurement for each tree.

After the parameters were predicted, the parameter values were graphed against stand variables, and correlation analysis and stepwise regression were performed. Tree factors tested for correlation included *DBH*, *HT*, stand age, *SD* and *SI*. Those factors and the combinations of power functions that were significantly related to the parameters were selected to establish Equation 5:

$$\zeta_j = f\left(HT_j, DBH_j, \ldots, A, SI, SD\right) \quad \ldots(5)$$

Where, HT_j is total height of the jth tree, DBH_j is the diameter at the breast of the jth tree, A is the stand age, SI is stand site index, and SD is initial stand density.

Third, according to the established function ζ_j, the related stand factors were introduced into the optimal model selected to construct the new model by the reparameterization method. With the best predictor variables determined, the different grouped data were refit to the three new constructed models. The best models with selected stand variables were then used as the NLME base models. All of these calculations were performed in S-Plus software (Insightful Corporation 2007).

Non-linear Mixed-Effect Modelling

In this study, a single-level non-linear mixed effect model was used to develop the crown profile models of Chinese fir. The specific forms are as follows:

$$\begin{cases} y_{ij} = f\left(\varphi_{ij}, v_{ij}\right) + e_{ij} & i=1,\ldots,M;\ j=1,\ldots,ni \\ \varphi_{ij} = A_{ij}\beta + B_{ij}b_{ij} \\ b_i \sim N(0, D) \\ e_{ij} \sim N\left(0, \sigma^2 R_i\right) \end{cases} \quad \ldots(6)$$

Where, y_{ij} is the jth observation value of the ith group, M is the number of groups, n_i is the number of observations of the group, f is a function of parameter φ_{ij} and covariate vector v_{ij}; β is the $(p\times1)$ dimensional fixed effect vector, b_i is the $(q\times1)$ dimensional random effect vector with variance-covariance matrix D, A_{ij} and B_{ij} are corresponding design matrices, e_{ij} is the error term of the normal distribution, σ^2 is the variance, R_i is the variance-covariance matrix of block i, and D is the variance-covariance matrix of random effects.

This paper introduces the application of the non-linear mixed-effects model into crown profile modelling. The model was constructed as follows:

Step 1: The key step in the NLME is to construct the parameter effects. The model with different random parameter combinations was fitted, and the mixed model with the best fitting effect was selected by comparing the mixed model fitting statistics, specifically the Akaike Information Criterion (AIC), Bayes Information Criterion (BIC) and the -2 Logarithmic Likelihood value (-2LL). A better fitting effect should have a smaller AIC, a smaller BIC, and a larger logarithmic likelihood. To avoid too many parameterization problems, the likelihood ratio test (LRT) was used to select the optimal model with different numbers of parameters, and the model with fewer parameters and the more significant model was chosen as the optimal model. The calculation formulas of AIC and BIC are as follows:

$$AIC = 2LL + 2d \quad \ldots(7)$$

$$BIC=-2LL+d\ln(n) \quad \ldots(8)$$

Where, LL is log likelihood, d is the number of estimated parameters of the model, and n is the number of valid observations.

Step 2: The variance-covariance structure D of random effects was determined. The variance-covariance structure depends on the number of random parameters, and the variance covariance structure of the two random parameters (u, v) is as follows:

$$D=\begin{bmatrix}\sigma_u^2 & \sigma_{uv}\\ \sigma_{uv} & \sigma_v^2\end{bmatrix} \quad \ldots(9)$$

Where, σ_u^2 is the variance of the random parameter u, σ_v^2 is the variance of the random parameter v, and σ_{uv} is the covariance of the random parameters u and v.

In this study, the random effect variance-covariance structure D was determined by comparing the fitting effects of three variance-covariance measures: the compound symmetry structure (CS), the diagonal matrix (DM) and the positive-definite matrix (PDM). Matrix forms with small AIC and BIC and large logarithmic likelihoods were selected.

Step 3: The intra-group variance-covariance structures R_i were determined to solve the problem of intra-group error correlation and heteroscedasticity. In forestry research, R_i are calculated by Equation 10 (Davidian & Giltinan 1995). The heteroscedasticity of a mixed model is often described by an exponential function (Equation 11) or power function (Equation 12). The application of correlation structure in mixed models is mainly used to simulate the correlation between intra-group errors. The commonly used correlation structures in forestry are autoregressive (AR), moving average (MA) and autoregressive-moving-average (ARMA). AIC, BIC and logarithmic likelihood were used to compare the effects of the correlation structures, and residual distribution was used to test the elimination of heteroscedasticity.

$$R_i = \sigma^2 G_i^{0.5} \Gamma_i G_i^{0.5} \qquad \ldots(10)$$

Where, σ^2 is the error variance value of the model, Γ_i is the intra-group error correlation structure, and G_i is the diagonal matrix describing the heterogeneity of variance.

$$g\left(u_{ij}, \alpha\right) = \left|u_{ij}\right|^{\alpha} \qquad \ldots(11)$$

$$g\left(u_{ij}, \beta\right) = \exp\left(\beta u_{ij}\right) \qquad \ldots(12)$$

Where, u_{ij} is a predicted value based on fixed effect parameters and α and β are the function parameters.

Model Validation

To evaluate the accuracy of the models, Fitting-effect statistical indicators selected were the determination coefficient (R^2, Equation 13) and the mean squared error (MSE, Equation 14). Statistical indicators used to evaluate the models were the Mean Deviation (MD, Equation 15), Root Mean Squared Error (RMSE, Equation16), the Mean Absolute Deviation (MAE, Equation 17), the AIC (Equation 18) and the BIC (Equation 19).

$$R^2 = 1 - \frac{\sum_{i=1}^{m} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{m} (y_i - \overline{y_i})^2} \qquad \ldots(13)$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad \ldots(14)$$

$$MD = \frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)}{n} \qquad \ldots(15)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2}{n-1}} \qquad \ldots(16)$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} \left| y_i - \hat{y}_i \right| \qquad \ldots(17)$$

$$AIC = n * \ln\left(MSE\right) + 2k \qquad \ldots(18)$$

$$BIC = n * \ln\left(MSE\right) + k \ln\left(n\right) \qquad \ldots(19)$$

where y_i represents the observed value for the i^{th} analytic tree, $\hat{y}_i$ is the predicted value of the i^{th} observed value, m is the number of observations, n is the number of observations for testing the equation, $\bar{y}_i$ is the mean value of the observations, and k is the number of parameters.

The test of the fixed effect part in the mixed effect model is the same as that of the traditional method but the test of the random effect part is to calculate the random parameter value through the second sampling; the random parameter value(b_k) is calculated according to the Vonesh & Chinchillas (1997) method:

$$\hat{b}_k \approx \hat{D} \hat{Z}_k^T \left(\hat{Z}_k \hat{D} \hat{Z}_k^T + \hat{R}_k \right)^{-1} \hat{e}_k \qquad \ldots(20)$$

Where, $\hat{D}$ is the variance-covariance matrix of random effect parameters, $\hat{R}_k$ is the intra-group variance-covariance structure, $\hat{Z}_k$ is the design matrix, and $\hat{e}_k$ is calculated by the actual value minus the predicted value calculated by the fixed effect parameter.

RESULTS

ANOVA and Multiple Comparisons

The results of LCR and LCL variance analysis (Table 3) indicated that there were significant differences in LCR and LCL under different stand ages, stand densities and site indexes at the significance level of 0.05. The results of the stand age tests are extremely significant.

Multiple comparative classification results showed that the number of LCR and LCL classifications under different site indexes were 3 and 5, respectively. The numbers of LCR and LCL classifications under different stand densities were 4 and 6, respectively. The numbers of LCR and LCL classifications under different stand ages were 3 and 3, respectively. From the results of multiple comparisons, it is not satisfactory to classify the modelling data according to site and stand density. Therefore, the modelling data were grouped by stand age. Since there was no significant difference between nearly mature forest and mature forest, they were uniformly classified as the same group. Finally, the crown data were classified into three groups according to stand age: Group I (young forest), Group II (medium forest) and Group III (nearly mature and mature forest).

Table 3: The variance analysis of LCR and LCL under different stand conditions.

Factor	Variable	Multiple Classification Number	F value	P value
Site index	LCR	3	2.313	0.036
	CL	5	17.941	<0.000
Stand density	LCR	4	2.953	0.008
	CL	6	6.341	<0.000
Stand age	LCR	3	57.343	<0.000
	CL	3	7.212	<0.000

Development of the Optimal Crown Profile Models

By comparing the fitting results of 24 basic crown profile models (Table 2), model-1(No.1, R^2=0.7582, MSE=0.0762) had the best fitting results for young forest (Group I). The optimal basic crown profile models of medium forest (Group II) and mature forest (Group III) were model-5 (No. 5, R^2=0.8013, MSE=0.0969) and model-20 (No. 20, R^2=0.9015, MSE=0.0195), respectively.

According to the correlation test between the parameters and the tree variables, the tree variables or combinations with strong correlation among the parameters were introduced into the model to reduce the collinearity between variables and ensure the accuracy of the model. The results showed that model-1 was not correlated with the stand variables and that model-5 and model-20 were most correlated with the SD.

The optimal crown profile models were selected for Groups I-III by the reparameterization method (Equations 21-23). Table 4 shows the model fitting and model validation results. These models passed the F-test and t-test. Moreover, the validation index values of MAE (0.0480~0.0950), absolute value of MD (less than 0.03) and RMSE (0.0969~0.1406) were small. This showed that the predicted values of the models did not differ from the actual values in the testing dataset.

$$CR = LCR\left(1 - RCH\right)^{a_0} \quad \ldots(21)$$

$$CR = LCR\left(b_0 \cdot \lg\left(N\right)\left(\frac{RCH-1}{RCH+1}\right) + b_1\left(RCH-1\right)\right) \quad (22)$$

$$CR = LCR\left(e^{c_0 + c_1 RCH + \left(c_2 \cdot \lg\left(N\right)\right)RCH^2}\right) \quad \ldots(23)$$

In Equations 21-23, a_0 and a_1 are model parameters, and *lg* is the base 10 logarithm.

Non-linear Mixed-effect Modelling

Construction of the parametric effects: In previous research methods, all parameters and combinations of the models were fitted as random effects. Based on the results of the statistical indices (AIC, BIC and LL) (Table 5), the mixed effect model showed a better performance than the NLS method. For Group I, the AIC and BIC of the mixed effect model were significantly lower than those of the basic model. For Group II, when considering a random parameter, model 2-2 was better than model 2-1 and the basic model. The significance of likelihood ratio test for model 2-3 and model 2-2 showed that adding one more random parameter significantly improved the fitting accuracy of the model. Model 2-3 showed the best performance, with an *AIC* of -137.6726, a BIC of -111.7462, and an LL of 75.83631. For Group III, when considering a random parameter, the model 3-1 had the smallest AIC and BIC. When adding one random parameter, model 3-4 was better than the other models. When considering 3 random parameters, the model 3-7 fitting did not converge.

Table 4: Model fitting and validation results of the three optimal crown profile models.

Age group	Parameter Estimates				Model fitting		Validation		
	Variable	Estimate value	S.E	P-value	R^2	MSE	MAE	RMSE	MD
Group I	a_0	0.6515	0.0130	<.0001	0.9015	0.0195	0.0950	0.1406	-0.0332
Group II	b_0	0.4314	0.0287	<.0001	0.8794	0.0443	0.0703	0.1398	0.0208
	b_1	-2.2975	0.0795	<.0001					
Group III	c_0	-0.1299	0.0207	<.0001	0.8930	0.0354	0.0480	0.0969	0.0156
	c_1	0.2727	0.1284	0.0342					
	c_2	-0.5802	0.0440	<.0001					

Construction of the variance-covariance structure: The fitting results of the variance covariance matrix are as follows. Because the model of Group I has only one parameter, the variance-covariance structure was not considered. For Group II, the fitting results of mixed models based on three different variance covariance structures were compared (Table 6), and the diagonal matrix had the smallest AIC and BIC values and the largest LL for both Group II and Group III. Therefore, in this study the diagonal matrix was taken as the variance-covariance matrix hypothesis form of random effects for medium forests and mature forests.

Autocorrelation and heteroscedasticity: To account for within-profile autocorrelation, the three correlation structures (*corAR1, corARMA* and *corCAR1*) were added to each crown profile model. The results of model fitting are given in Table 7. We concluded that for Group I and Group II, the *corARMA* structure had the most obvious improvement on the fitting effect of mixed models. For Group III, the three evaluation indicators showed that the *corCAR1* model could significantly improve the accuracy of the model. Therefore, *corCAR1* was used to describe the sequence autocorrelation structure of the crown profile model.

In this paper, power functions and exponential functions were used to account for heteroscedasticity. The power function equation performed better in the mixed model for Group I, whereas the exponential function was better than the power function for Group II and Group III.

In addition, plots of weighted residuals against predicted values for crown profile models (Fig. 2 (a-c)) showed that the residual values of the three models were uniformly distributed above and below the 0-level line. This indicated that the heteroscedasticity functions used to estimate weights were effective.

Table 8 gives the comparison of the fitting results between the mixed model and the traditional model. The

Table 5: Model fitting results based on different random effect parameters.

Age group	Model	Random effect Parameter	AIC	BIC	LL	LRT	P-value
Group I	Basic model	---	-415.5835	-407.7031	209.7917	-	-
	Model1-1	a_0	-523.0991	-511.2786	264.5496		
	Basic model	---	-79.5781	-68.4667	42.7891	-	-
Group II	Model 2-1	b_0	-127.2645	-112.4494	67.6323	-	-
	Model 2-2	b_1	-129.3998	-114.5847	68.6999	51.8218	<.0001
	Model 2-3	b_0, b_1	-137.6726	-111.7462	75.8363	14.2728	0.0026
	Basic model	---	-221.0012	-204.5642	114.5006	-	-
	Model 3-1	c_0	-327.2169	-306.6707	168.6084	108.2157	<.0001
Group III	Model 3-2	c_1	-298.9860	-278.4397	154.4930	-	-
	Model 3-3	c_2	-243.5823	-223.0361	126.7912	-	-
	Model 3-4	c_0, c_1	-336.5379	-307.7731	175.2689	17.3210	<.0001
	Model 3-5	c_0, c_2	-330.0363	-301.2716	172.0181	-	-
	Model 3-6	c_1, c_2	-319.0273	-290.2625	166.5136	-	-
	Model 3-7	c_1, c_2, c_3	--	--	--		

Table 6: Comparison of fitting results of mixed models with different variance covariance structures based on the random parameter effect.

Age group	Variance-covariance structure	AIC	BIC	LL
Group II	PDM	-127.3548	-105.1321	69.6774
	DM	-127.4096	-108.8907	68.7048
	CS	-126.1750	-107.6561	68.0875
Group III	PDM	-336.5379	-307.7731	175.2689
	DM	-338.3814	-313.7259	175.1907
	CS	-333.0058	-308.3503	172.5029

fitting results indicated that the R^2 values of the 3 basic models were 0.9015, 0.8794 and 0.8930 and that the RMSEs of the 3 basic models were 0.1395, 0.2102 and 0.1878, respectively. The R^2 values of the 3 mixed models were 0.9214, 0.9129 and 0.9398, and the RMSEs of the 3 mixed models were 0.1246, 0.1786 and 0.1409, respectively. Therefore, all accuracies of the crown profile models were improved by adding autocorrelation structures and the heteroscedasticity function. Therefore, the NLME modelling was selected to construct the crown models for Chinese fir.

Model Validations

The basic model and the mixed effect model were validated with independent testing datasets. Equation 20 was used to calculate the random parameters, and the detailed calculation procedure was performed with S-Plus software. The MD, MAE and RMSE values of the 3 indexes were used to compare the prediction accuracy with the traditional NLS method. The validation results are shown in Table 9.

We can see that the MD, MAE and RMSE values of the mixed model performed better than those of the basic

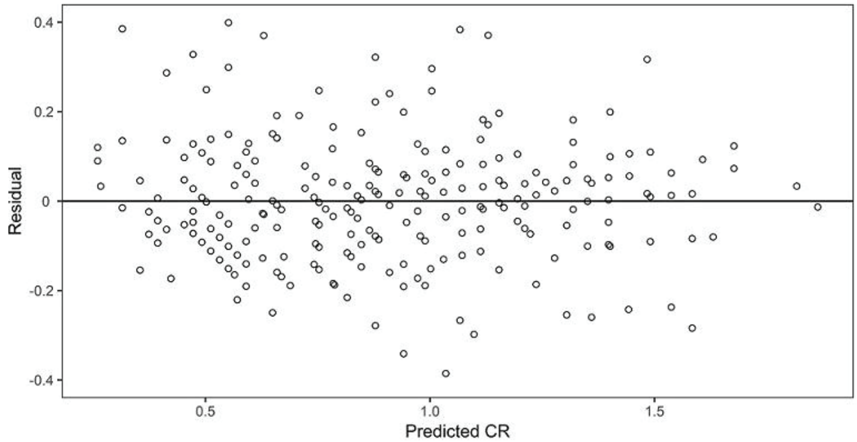
Table 7: Comparison of fitting results of mixed model based on different autocorrelation error structures.

Age group	Autocorrelation Error Structure	AIC	BIC	LL	LRT	P
Group I	No	-523.0991	-511.2786	264.5496		
	corAR1	-559.3740	-539.6731	284.6870	40.2748	<.0001
	corARMA	-561.7973	-538.1563	286.8987	44.6982	<.0001
	corCAR1	-555.8533	-536.1524	282.9266	36.7542	<.0001
Group II	No	-129.3998	-114.5847	68.6999		
	corAR1	-167.7065	-141.7800	90.8532	44.3067	<.0001
	corARMA	-175.5464	-145.9161	95.7732	54.1466	<.0001
	corCAR1	-166.3127	-140.3862	90.1563	42.9128	<.0001
Group III	No	-338.3814	-313.7259	175.1907		
	corAR1	-420.5575	-387.6835	218.2788	86.1761	<.0001
	corARMA	-394.7720	-357.8489	206.3860	79.5759	<.0001
	corCAR1	-423.0464	-390.1725	219.5232	88.6651	<.0001

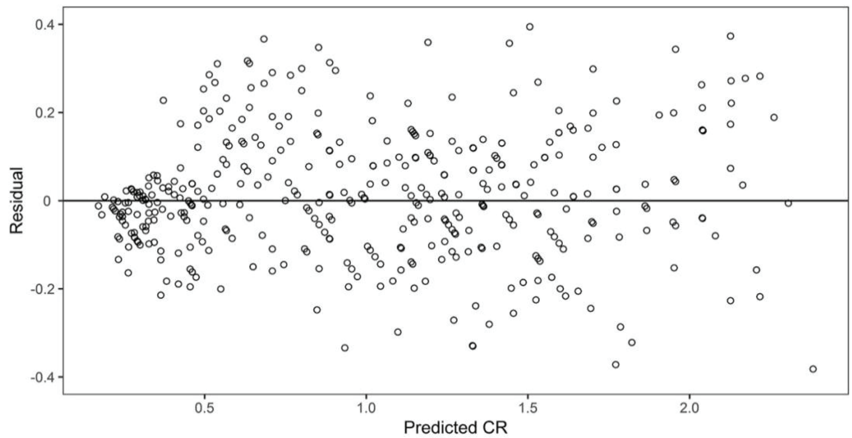
Table 8: Fitting results between basic model and mixed model based on different variance functions.

Items	Parameters	Group I		Group II		Group III	
		Basic Model	Mixed Model	Basic Model	Mixed Model	Basic Model	Mixed Model
Fixed parameter	$a_0(b_0, c_0)$	0.6516	0.6573	0.4314	0.5285	-0.1297	-0.1484
	$a_1(b_1, c_1)$			-2.2975	-2.5584	0.2713	0.5123
	$a_2(c_2)$					-0.5797	-0.6690
	a_3						
Variance component	δ^2	0.0195	0.0215	0.0443	0.0100	0.0354	4.3708e-003
	δ^{2a0}		0.0064		0.0009		9.2022e-011
	δ^{2a1}				0.0002		5.3092e-002
	α		0.5491				
	β				0.4987		0.6564
Fitting indices	R^2	0.9015	0.9214	0.8794	0.9129	0.8930	0.9398
	RMSE	0.1395	0.1246	0.2102	0.1786	0.1878	0.1409

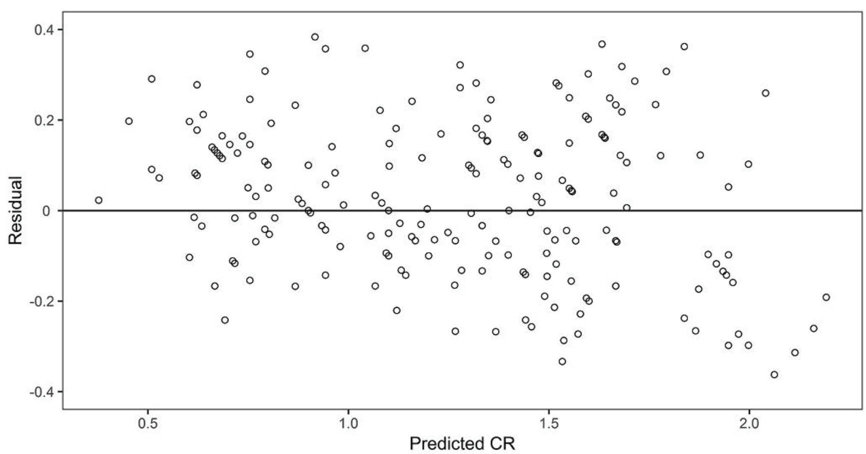
Note: α, β are the parameters of power function and exponential function.



(a)



(b)



(c)

Fig. 2: Distribution of residuals for three equations predicting the crown radius of Chinese fir trees.

model. These evaluation indexes showed that incorporating random parameters significantly improved the prediction accuracy of the model.

Table 9: Validation results of crown profile models.

Age group	Model	MAE	RMSE	MD
Group I	Basic model	0.0950	0.1406	-0.0332
	mixed-effect model	0.0809	0.1008	-0.0167
Group II	Basic model	0.0703	0.1398	0.0208
	mixed-effect model	0.0514	0.1016	0.0154
Group III	Basic model	0.0480	0.0969	0.0156
	mixed-effect model	0.0214	0.0612	0.0023

DISCUSSION AND CONCLUSIONS

Tree crown shape changes regularly with changes of stand age, stand density and site index. To construct a crown profile model more accurately and improve the accuracy of the model, crown data were classified and fitted by single factor ANOVA and multiple comparison methods based on the crown survey data of Chinese fir in Fujian Province. We used age, density and site index as the objects of variance analysis to study whether these three stand factors have significant effects on LCR and CL and to classify the data by LSD difference results. The results showed that the model based on the data of different age groups was reasonable. Therefore, the modelling data were classified into three groups after the analysis of variance and multiple comparisons: I (young forest), II (medium forest), and III (nearly mature and mature forest). In our study, 24 alternative equations of crown profile models were compared.

To explore the crown shape, stand factors (density, site) and tree factor variables such as DBH and tree height, the relationships between stand factor variables were introduced in this paper into the previous optimal model by using the reparameterization method, and three new models of different age groups were developed. For Group I, the simple single equation ($CR = LCR(1-RCH)^{a_0}$) was selected, which showed that the stand variable had little effect on the model parameters. The crown profile models of Group II and Group III were $CR = LCR\left(e^{b_0+b_1RCH+(b_2\lg(N))RCH^2}\right)$ and $CR = LCR\left(c_0\lg(N)\left(\frac{RCH-1}{RCH+1}\right)+c_1(RCH-1)\right)$, respectively; and the two models were most correlated with the stand density. Therefore, the influence of stand variables on crown shape was considered in model construction, and the models constructed were more reasonable.

In this study, a single-level non-linear mixed effect model was used to develop the crown profile model of Chinese fir. To account for within-profile autocorrelation, three correlation structures (*corAR1, corARMA* and *corCAR1*) were added to each crown profile model. The structure of the power function and exponential function were used to account for heteroscedasticity. Compared with the NLS, the mixed effect model could account for the autocorrelation of crown data and performed better. The NLME model reflects the difference of crown radius at different positions in the same crown by random parameters, whereas the traditional NLS can only reflect the average change rule of crown radius. Due to data constraints, the NLME model in this study only considered the effects of repeated measurements in single-level trees. This study did not consider both sample plot and single tree level at the same time, although the accuracy of NLME may be higher if the two levels of sample plot and single trees are considered. In the future, we need to continue study on the basis of increasing sample size.

To vividly describe the crown shape of Chinese fir under different growth conditions, the crown profile models under three different age groups determined by the method of non-linear mixed effect were used to display the crowns using 2D and 3D graphics-rendering technology in Fig. 3(a-c) and Fig. 4(a-c). Fig. 3(a-c) shows 2D tree crown maps of three different groups under certain stand growth conditions. Fig. 4(a-c) shows the 3D tree crown enveloping surfaces of the corresponding groups. It can be seen that the crown shape changes with age. The height of young Chinese fir trees rapidly increases, and the lateral branches are underdeveloped. Competitive trees do not affect the crown shape. The crown shape of young Chinese fir trees is nearly conical. In the middle-aged forest stage, the growth of tree height gradually tends to be flat, the growth of lateral branches is relatively developed, and the position of maximum crown radius gradually moves up. In the mature forest stage, tree height growth is slower, and the surrounding trees impact a subject tree. The growth of lateral branches at the head of crown tips often exceeds that of the main tips, and the crown shape gradually trends to a flat top. Therefore, the crown shape can be well described by an exponential-form equation at this time.

Many forms of crown profile models have been used to simulate different tree species. However, there are few studies of crown profile models of the same tree species in different periods. The expressions of the crown profile models constructed in this study are clear and simple, and the models have substantial application value. The fitted crown profile models can be applied with only LCR, LCL and N values, which can easily be found in actual forest surveys.

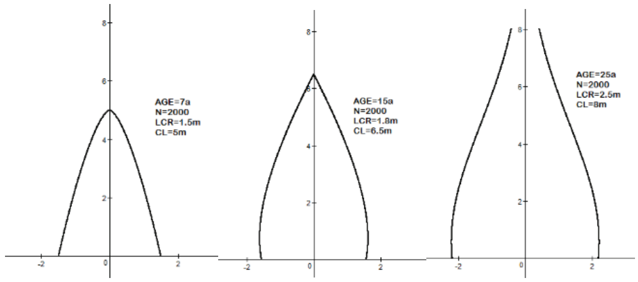


Fig. 3: 2D Crown shapes in different years. (a) Group I: AGE = 7 year, N = 2000, LCR =1.5 m, CL = 5 m; (b) Group II: AGE = 15 year, N = 2000, LC R = 1.8 m, CL = 6.5 m; (c) Group III: AG E= 25 year, N = 2000, LCR = 2.5 m, CL = 8 m.

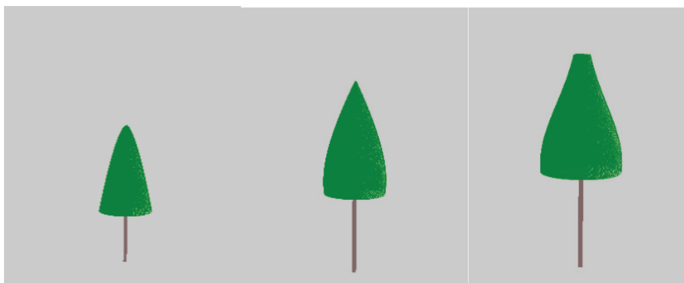


Fig. 4: 3D Crown shapes in different years. (a) Group I: AGE = 7 year, N = 2000, LCR =1.5 m, CL = 5 m; (b) Group II: AGE = 15 year, N = 2000, LCR = 1.8 m, CL = 6.5 m; (c) Group III: AGE = 25 year, N = 2000, LCR = 2.5 m, CL = 8 m.

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