



Study on Water Environmental Carrying Capacity Based on Improved Catastrophe Theory

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ABSTRACT

The improved catastrophe theory is introduced into the study of water environment carrying capacity. With coupling of 4 subsystems which are, water resource factors, ecological environment factors, social resources allocation capacity factors, social and economic factors as the basic framework, 12 indexes were selected to construct the model of water environment carrying capacity. Based on the characteristics of normalization formula of aggregation, the rating scale refinement method was used to improve conventional catastrophe theory and to obtain the adjusted comprehensive value. Then according to the evaluation habits, fuzzy judgment was selected to determine the evaluation threshold, five grade evaluation was used to evaluate water environment carrying capacity. At the last, the water environment carrying capacity in Yichang city was evaluated, the results show that: during 2010-2015, water environmental carrying capacity in Yichang city was on the rise, but in 2020 the Yichang city water environmental carrying capacity will decrease sharply. We need to strengthen environmental protection and governance level.

INTRODUCTION

Water environmental carrying capacity is a supporting capacity of the water body to human activities under the conditions of a particular productivity status and meeting the specific environmental objectives, and the premise of self-sustaining, self-regulating and ensuring the sustainable development (Bai et al. 2016, Li et al. 2014a, Yang & Tong 2016). It inherits the core connotation of sustainable development, and measures the degree of coordination between water environment and social economic development. Therefore, under the current situation of population explosion, rapid economic development, and serious contradiction between water resource exploitation and the ecological environment protection, the research on water environment carrying capacity becomes particularly important for ensuring the coordinated development of regional human activities and environment and scientific formulation of water environmental protection policy (Wang et al. 2007).

At present, the research on the quantitative methods of water environment carrying capacity is still in the exploratory stage, the existing methods have their own advantages and disadvantages, and the academic circles have not yet formed a unified and accepted quantitative method of water environment carrying capacity (Sun et al. 2015).

The commonly used methods include: (1) Index system evaluation method, its quantitative models mainly include: vector mode method (Shen et al. 2015), fuzzy comprehensive evaluation method (Geng 2012), principal component analysis method, etc., but they still depend on the human judgment, which is with subjective arbitrary; (2) The system dynamics method (Jiao et al. 2015, Wang et al. 2009, Xie et al. 2012) can be used to simulate the system dynamics model for different development plans, and predict the decision variables, but its parameter is not easy to master, and it is easy to lead to unreasonable conclusion; (3) Multi objective model optimization method (Wang & Chen 2010) uses decomposition-coordination system analysis principle, decomposes the whole research system into several subsystems, and uses the mathematical model to describe. Its applicability is reduced, because it needs large demand of data and the model is difficult to solve with large computation.

In view of this, the catastrophe theory is introduced into the field of quantification of water environment carrying capacity in this paper, and aim at the characteristic of aggregation and large of the calculation results of the conventional catastrophe evaluation method, the rating scale refinement method is adopted to adjust the initial comprehensive value. Finally, according to the human evaluation

habits, the evaluation grade is generalized to five grade fuzzy judgment, the corresponding evaluation threshold is determined to make the final comprehensive evaluation.

PRINCIPLE AND METHOD OF CATASTROPHE THEORY

Catastrophe theory is a scientific study on the characteristics of non continuity and sudden change of objective things, which was found by French mathematician Thom Rene (Li et al. 2014b, Shao et al. 2012) in 1970s. Catastrophe theory can directly deal with the discontinuity, without the relation to any special internal mechanism, which is especially suitable for the research on the system of which the internal action has not yet determined (Poston & Stewart 1978). The mathematical model of this theory is simple, and it can solve the unknown problem that the traditional mathematical method cannot solve, so its field of application is wide. Table 1 gives several commonly used catastrophe models.

IMPROVED CATASTROPHE EVALUATION METHOD OF WATER ENVIRONMENTAL CARRYING CAPACITY

Multi criteria evaluation method of catastrophe theory: Catastrophe evaluation method is a multi criteria evaluation method based on catastrophe theory, its main steps are:

Step 1: Construction of the evaluation index system. According to the internal mechanism of the system, reasonably choose the indicators, establish multi-level index tree system, and array the indicators in order of their importance.

Step 2: Standardization of the original data of the base indicators (control variables). The original data of each index are normalized to obtain a set of non dimensional initial membership value in [0, 1].

For the index that the larger the more optimal:

$$x = (x_i - x_{\min}) / (x_{\max} - x_{\min}) \quad \dots(1)$$

For the index that the smaller the more optimal:

$$x = (x_{\max} - x_i) / (x_{\max} - x_{\min}) \quad \dots(2)$$

Step 3: Normalized calculation. Use unitary formula to implement comprehensive quantitative calculation, recursive calculation to obtain the system’s initial comprehensive evaluation value R_j . Research the nature of the issues according to the actuality, follow the “complementary” or “non complementary” criteria.

(1) Non complementary criterion: When the role of control variables to state variables cannot be replaced with each other, get values according to minimax criterion.

(2) Complementary criterion: When the role of control variables to state variables can complement each other, use its mean value.

Step 4: Repeat Step 1-3. Calculate the total catastrophe membership function value, and take it as a comprehensive evaluation value.

Improvement of catastrophe evaluation method: The most obvious defect of catastrophe evaluation method is from the characteristic of aggregation of unitary formula, the final calculated comprehensive evaluation values are high (close to 1), and the difference between the evaluation values is very small, which is difficult to convince, and not conducive to human’s intuitive judgment, easy to make people misunderstand (Shi et al. 2003). In order to solve this problem, the method of adjusting the initial comprehensive value (which takes the comprehensive evaluation value obtained via the catastrophe evaluation method, as the initial value) is adopted to transform the comprehensive evaluation value calculated by unitary formula into an adjustment comprehensive value. The steps are as follows:

Step 1: Refine the rating scale. Respectively, calculate the top catastrophe evaluation value r_n when the base catastrophe evaluation value is selected from {0, 0.05, 0.1, 0.15, ..., 0.95, 1} and they are used as the rating scales to describe the comprehensive value of the general catastrophe evaluation,

Table 1: Common catastrophe model.

Catastrophe model	Control variable	Potential function	Normalization formula
Fold catastrophe	1	$f(x) = x^3 + ax$	$x_a = \sqrt{a}$
Cusp catastrophe	2	$f(x) = x^4 + ax^2 + bx$	$x_a = \sqrt{a}, x_b = \sqrt[3]{b}$
Swallowtail catastrophe	3	$f(x) = x^5 + ax^3 + bx^2 + cx$	$x_a = \sqrt{a}, x_b = \sqrt[3]{b}, x_c = \sqrt[4]{c}$
Butterfly catastrophe	4	$f(x) = x^6 + ax^4 + bx^3 + cx^2 + dx$	$x_a = \sqrt{a}, x_b = \sqrt[3]{b}, x_c = \sqrt[4]{c}, x_d = \sqrt[5]{d}$

the corresponding interval of different grades is ; $[r_n, r_{n+1}]$ ($n = 1,2,3,\dots,19$).

Step 2: Take the comprehensive evaluation value calculated by the catastrophe evaluation method as the initial value R_i , then according to the rating grade $[r_n, r_{n+1}]$ ($n = 1,2,3,\dots,19$) it belongs to, map it to the corresponding uniform interval, according to the formula (3). Obtain the initial comprehensive evaluation value R'_i , which is the adjusted comprehensive value derived from the improved catastrophe evaluation method.

$$R'_i = 0.05 \left[\left(\frac{R_i - r_n}{r_{n+1} - r_n} \right) + n \right] \dots(3)$$

Step 3: Evaluate the comprehensive evaluation values based on adjusted comprehensive values, the greater the adjusted comprehensive value is, the better the solution is.

EVALUATION OF WATER ENVIRONMENTAL CARRYING CAPACITY BASED ON IMPROVED CATASTROPHE THEORY

Yichang city is located in the southwest of Hubei Province, the nodes between upstream and midstream of Yangtze River, intersection zone of Chongqing Hubei and Hunan. The geographical environment of Yichang is complex and diverse, and the geological structure is complex, low-west high-east, there are mountains, plains and hills in Yichang. Yichang is located in the transition zone of tropical and sub tropical, subtropical monsoon climate, with subtropical monsoon humid climate, and has the climatic characteristics of four distinctive seasons, water and heat in the same season, and cold and dry in the same season. The annual

average rainfall is 1215.6 mm, and the average temperature is 16.9°C.

Yichang has 3 county-level cities, five counties, five districts, a total population of 4 million and 150 thousand people, the land area is 21 thousand square kilometres, and the urban area is 4249 square kilometres. Yichang is an important transportation hub in the central region of China, Yangtze River route, 3 railways, and 5 highways throughout the area. The Three Gorges Dam, Gezhouba Dam and other hydropower facilities are a strong economic support of the area. Comprehensive strength ranked second in Hubei for many years. Due to the special geographical location, it is also known as the sub-centre city of Hubei province.

Construction of index system: The index of water environment carrying capacity is numerous, and has many interactional factors, and the construction of evaluation index system is complex. In this paper, referring to the relevant research results, the index system of water environmental carrying capacity is constructed from the consideration of the frequency of the use of indicators and the difficulty of quantitative indicators, which is shown in Fig. 1.

The index system includes the target layer (A), the criterion layer (B) and the index layer (C). Based on the evaluation criteria of water resources factor, ecological environment factor, social resource allocation capacity factor and social economic factor, the basic framework is constructed. It covers a total of 12 evaluation indicators, each layer index is arranged from the left to the right according to the influence degree on the index of the upper layer, which is basically able to reflect the status of regional water environment carrying capacity.

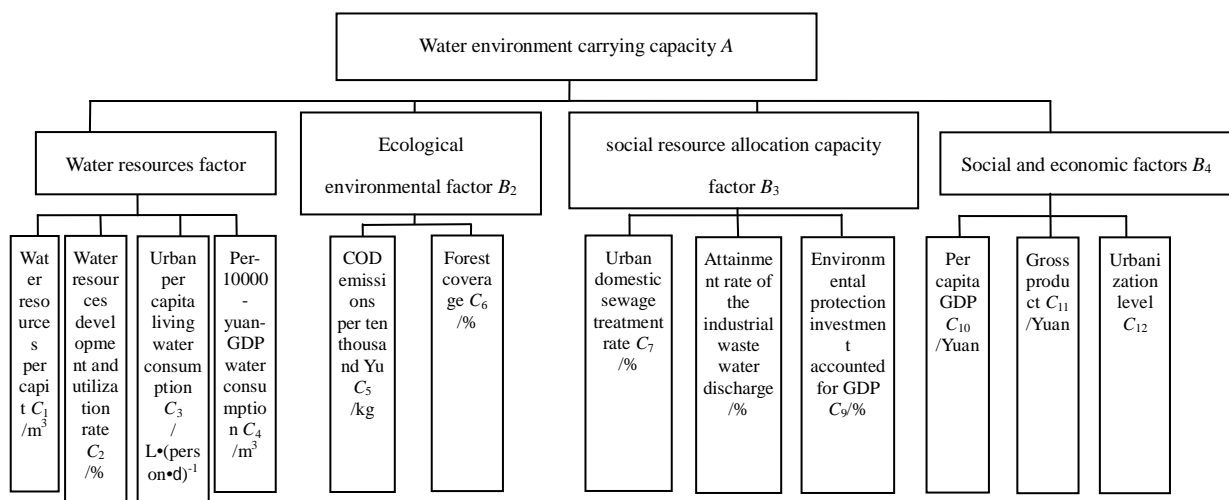


Fig. 1: Index system of water environmental carrying capacity.

Table 2: Water environment carrying capacity indexes value.

Category	Index title	Index property	Year		
			2010	2015	2020
Water resources factor B_1	Per capita water resources $C_1(m^3)$	(+)	3239	3171.2	2937.6
	Water resources development and utilization rate $C_2(\%)$	(-)	11.31	14.74	19.95
	Urban per capita living water consumption $C_3(L \cdot (person \cdot d)^{-1})$	(-)	162	177	192
	Per-10000-Yuan-GDP water consumption $C_4(m^3)$	(-)	98	55.43	52.5
Ecological environmental factor B_2	COD emissions per ten thousand Yuan $C_5(kg)$	(-)	0.88	0.85	0.82
	Forest coverage $C_6(\%)$	(+)	55.3	65.7	60
Social resource allocation capacity factor B_3	Urban domestic sewage treatment rate $C_7(\%)$	(+)	89.25	90	90
	Attainment rate of the industrial waste water $C_8(\%)$	(+)	99.73	99.73	99.73
Social and economic factor B_4	Environmental protection investment accounted for GDP (%)	(+)	2.05	2.05	2.05
	Per capita GDP $C_{10}(\text{Yuan})$	(+)	38520	82465.59	111610
	Gross product $C_{11}(\text{Yuan})$	(+)	1547.32	3384.80	6769.6
	Urbanization level C_{12}	(+)	0.32	0.54	0.6

Table 3: Rating scale refined the comprehensive value.

Degree of membership of base index	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
Grade n	0	1	2	3	4	5	6	7	8	9	10
Target layer threshold r_n	0	0.7049	0.7591	0.7934	0.8353	0.8558	0.8731	0.8881	0.9014	0.9133	0.9241
Degree of membership of base index	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1	
Grade n	11	12	13	14	15	16	17	18	19	20	
Target layer threshold r_n	0.9341	0.9433	0.9519	0.9599	0.9675	0.9747	0.9815	0.9879	0.9941	1	

Table 4: Results of water environment carrying capacity based on improved catastrophe theory.

Year	2010	2015	2020
Catastrophe evaluation method R_i	0.8103	0.9384	0.8073
Grade n	3	11	3
Improved catastrophe evaluation method R_i'	0.1702	0.5735	0.1666

Table 5: Water environment carrying capacity in five grades.

Water environmental carrying capacity level	Level I Poor	Level II Pass	Level III (2015) medium	Level IV Good	Level V excellent
Degree of membership of base index	(0,0.2)	(0.2,0.4)	(0.4,0.6)	(0.6,0.8)	(0.8,1)
Threshold of catastrophe theory	(0,0.8353)	(0.8353,0.9014)	(0.9014,0.9433)	(0.9433,0.9747)	(0.9747,1)
Threshold of improved catastrophe theory	(0,0.3)	(0.3,0.5)	(0.5,0.7)	(0.7,0.9)	(0.9,1)

Basic data standardization: According to 2010 and 2015 “Yichang Statistical Yearbook”, “water resources bulletin”, “environmental bulletin”, “government work report” and other information, the index values of water resources carrying capacity in 2010 and 2015 are obtained. According to the “planning of Yichang in 12th Five-Year”, the forecast values of the indicators in 2020 are obtained as given in Table 2.

The property of the index in the table indicates the positive and negative property of the index. If the indicator is +

($C_1, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}$), the indicator is positive indicator, the larger the better; namely the greater the indicator is, the greater the water environment carrying capacity is. Carry out the original data standardization according to the formula (1). If the indicator is - (C_2, C_3, C_4, C_5), the indicator is negative indicator, the smaller the better; namely the smaller the indicator is, the greater the water environment carrying capacity is. Carry out the original data standardization according to the formula (2).

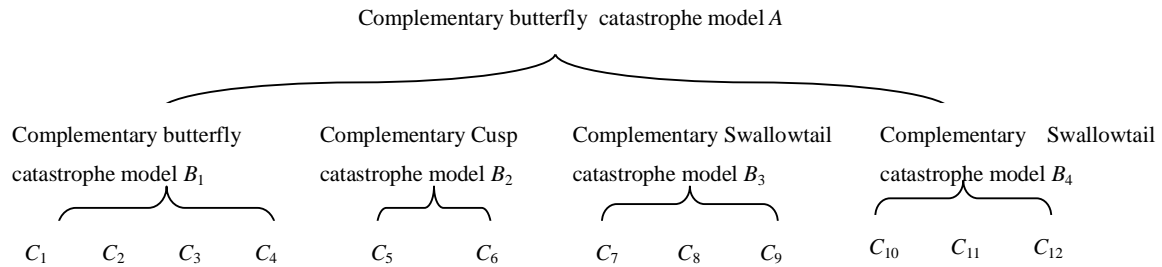


Fig. 2: Catastrophe model of water environment carrying capacity evaluation.

Improved catastrophe value calculation: In the index layer, complementary butterfly catastrophe model is composed of the index $C_1 \sim C_4$, complementary cusp catastrophe model is composed of the index $C_5 \sim C_6$, and complementary swallowtail catastrophe model is composed of the indexes $C_7 \sim C_9$ and $C_{10} \sim C_{12}$. In the criterion layer, complementary butterfly catastrophe model is composed of the index $B_1 \sim B_4$. Step by step calculate according to the unitary formula in Table 1, as shown in Fig. 2, to obtain the initial evaluation value.

Refine the rating scale, when the degree of membership of index layer is respectively, $\{0, 0.05, 0.1, 0.15, \dots, 0.95, 1\}$, calculate the target layer catastrophe threshold r_n , and it is used as the rating scale to describe the comprehensive value of the general catastrophe evaluation as given in Table 3. If the calculated initial evaluation value R_i is above $[r_n, r_{n+1}]$ ($n = 1, 2, 3, \dots, 19$), the water environment carrying capacity grade is regarded as grade $n+1$.

According to formula (3), transform the initial evaluation value R_i into adjustment value R'_i of comprehensive evaluation value, namely the adjusted comprehensive values derived from the improved catastrophe evaluation method, as indicated in Table 4.

Quantification of the five level threshold of water environmental carrying capacity evaluation: In life and work, people usually use the five levels to evaluate things, they are accustomed to use fuzzy judgments, such as excellent, good, medium, pass and poor, as the accounting standard. In the above, the improved catastrophe evaluation, for the accuracy of calculation, the water environment carrying capacity is divided into 20 levels, which is not conducive to draw a direct conclusion that is easy to be accepted by people.

In order to determine the status of water environmental carrying capacity more intuitively, the water environment carrying capacity is divided into five levels. According to the method of selecting the uniform value of the base index, the classification limits of the 5 grades are obtained, $x_1=0$,

$x_2=0.2, x_3=0.4, x_4=0.6, x_5=0.8, x_6=1$, and calculate the five grades threshold in accordance with the improved catastrophe method. The target layer is also divided into 5 sections, as given in Table 5.

The calculation results by the improved catastrophe method show that, $R'_{2010} = 0.1702, R'_{2020} = 0.1666$, namely, the water environment carrying capacity is poor (grade I) in 2010 and 2020, which means that the regional human activities and water resources, water environment are in a serious state of non coordinated development. We need to increase the degree of environmental protection and management efforts; $R'_{2015} = 0.5735$, which means that the water environment carrying capacity is in the middle level (III) in 2015, and the human activities and the water resources, and the water environment are in a general coordinated development state. Human activities will not only cause harm to the environment, but it also will not have a positive effect on the environment. If we do not pay attention and strengthen the protection of water environment in the further development of society, it will be more and more severe.

During 2010-2015, water environmental carrying capacity in Yichang is on the rise, but in 2020 the Yichang water environmental carrying capacity will decrease sharply. Combined with the basic data in Table 2, the reasons for it are analysed as: with the development of economy and society of Yichang, the population increases, and the quantity of water resources for the population will decrease significantly, but the water consumption will continue to rise. The development and utilization of water resources will increase, resulting in the situation that the water environment is tight and the water environment carrying capacity decreases.

CONCLUSIONS

Evaluation of water environmental carrying capacity is a multi criteria and multi hierarchy comprehensive evaluation. In this study, the improved catastrophe theory was introduced into the study of water environmental carrying

capacity, and the index system of water environmental carrying capacity is established, the discrimination threshold is determined, and a mature water environment carrying capacity research system is formed.

1. The application of catastrophe theory to the evaluation of water environmental carrying capacity is not common. In this paper, with coupling of 4 subsystems which are water resources factors, ecological environment factors, social resources allocation capacity factors, social and economic factors as the basic framework, 12 indexes are selected to construct the model of water environment carrying capacity. This method only needs to consider the relative importance of the index, which avoids the subjectivity and complexity of the weight determination.
2. The evaluation values obtained by the catastrophe evaluation method are all larger than 0.8, and after calculating, the evaluation value in 2010 is 0.8103, but the evaluation results of water environmental carrying capacity is poor, which is not in conformity with human habits. It is difficult to convince, and not conducive to intuitive judgment. By using the rating scale refinement method to improve the conventional catastrophe theory to obtain the comprehensive values, can skilfully overcome the characteristics of aggregation and large calculation results of the conventional catastrophe evaluation method.
3. According to the human habits, "excellent, good, medium, pass and poor" are used as fuzzy judgment, the evaluation threshold is determined, five grade evaluation is applied to water environment carrying capacity evaluation, which is convenient to determine the regional water environmental carrying capacity situation more intuitively and more clearly.
4. Quantitative evaluation is implemented to water environmental carrying capacity of Yichang city in 2015 and 2010. According to the relevant planning data, the water environmental carrying capacity in 2020 of Yichang city is predicted. The results show that, in 2010, the water environment carrying capacity of Yichang city is low. With the continuous development of economic and social development in Yichang, and the increase of population, the water environment carrying capacity in Yichang city in 2020 is expected to dramatically reduce. We need to strengthen the environmental protection and governance level.

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