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Original Research Paper

Research on 2-D Ecological Mathematical Model of Red Tide

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ABSTRACT

The research uses the ADI (Alternating Direction Implicit) method to disperse and solve the two-dimensional shallow water circulation equations, based on upwind scheme; the red tide biodynamic equations are dispersed, then a two-dimensional ecological mathematical model of red tide is built by combining hydrodynamics with biodynamics. The two-dimensional ecological mathematical model of red tide is applied in Bohai Sea to simulate the actual *Phaeocystis* red tide instance which occurred on 6th-16th June, 2004. The calculated area of red tide from the mathematical model is compared with that from remote sensing photos of EOS/MODIS secondary planet. The results show that the model simulate the process of the increase and decrease of red tide well, which provides a scientific basis for a red tide forecast of Bohai Sea.

INTRODUCTION

Nowadays mankind is confronted with considerable water environmental problems, which engage the attention of a large number of researchers such as Di Toro et al. (1971), Atlas et al. (1972), Moore et al. (1984), Suess (1985) and Nagheeby & Kolahdoozan (2010). Research on red tide has been over 40 years, from the red tide organism physiological and biochemical research, to the mechanism of red tide and red tide prediction research, involving more deeper field (Takahashi et al. 1982, Fukuju et al. 1998, Walsh et al. 2001, Lenes et al. 2008). The origin of red tide is very complex; in addition to its own characteristics, it also involves physics, chemistry, hydrology, meteorology and many other factors. So using hydrodynamics to integrate with every process of red tide is imperative to further research (Kamykowski 1981, Kishi et al. 1989, Yanagi et al. 1995, Andrew & John 1996, Lee et al. 2004). In China, Xia Zongwan et al. (1997) set up the ecological simulation model of red tide that included water and biodynamics. Tian Feng et al. (2007) set up a depth model of growth of nutrient phytoplankton by combining dynamics with hydrodynamics. Chen Xiuhua et al. (2007) established a three-dimensional baroclinic model for the summer of East China Sea. Wang Hongli (2002) made a nonlinear dynamics research on the algal model in Bohai Sea.

To achieve a reliable prediction of red tide, numerical simulation is important which can combine all kinds of dynamic process, clarify the quantitative relations from dynamics theory of nutrient biogeochemical process and the key physical processes of the red tide. This paper sets up a two-dimensional ecological mathematical model of red tides built up by combining hydrodynamics with biodynamics, so that a reliable prediction of the red tide is more possible.

MATERIALS AND METHODS

Basic hydrodynamic control equations: The incompressible fluid two-dimensional shallow water circulation equations:

$$\frac{\partial \xi}{\partial t}+\frac{\partial((\xi+h)u)}{\partial x}+\frac{\partial((\xi+h)v)}{\partial y}=0 \qquad ...(1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial \xi}{\partial x}=-g\frac{\sqrt{u^2+v^2}}{c^2h}u+fv+A_H\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad ...(2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial \xi}{\partial y}=-g\frac{\sqrt{u^2+v^2}}{c^2h}v-fu+A_H\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad ...(3)$$

Where, ξ is added tide level; h is tide level; u is the average velocity in x direction; v is the average velocity in y direction; g is the gravitational acceleration; $c=\frac{1}{n}h^{\frac{1}{6}}$ is the Chezy coefficient, n is roughness; f is the Coriolis coefficient; A_H is horizontal eddy viscosity.

Disposal of boundary condition: Shore boundary: $v_n = 0$ (n is the direction of the border)

Water boundary: $\frac{\partial v}{\partial n}=0$; $\xi=\xi^*$

Shore boundary: when $t = 0$, then $\xi = 0$, $u = v = 0$

Model dispersion of ADI: ADI method Gustafsson (1971) and Li Daming (2012) is used to disperse the hydrodynamic equation; to simplify the following differential expression, the symbols are defined as follows:

$F_{i,j}^{(k)} = F(i\Delta x, j\Delta y, k\Delta t)$, $\Delta x = \Delta y = \Delta s$,

Where,

$i = 0, \pm 1/2, \pm 1, \pm 3/2, \cdots\cdots$; $j = 0, \pm 1/2, \pm 1, \pm 3/2, \cdots\cdots$;

$k = 0, 1/2, 1, 3/2, 2, \cdots\cdots$;

$$\overline{F}^{y}_{i,j+1/2} = \tfrac{1}{2}(F_{i,j} + F_{i,j+1});$$

$$\overline{F}^{y}_{i,j+1/2} = \tfrac{1}{2}(F_{i,j} + F_{i,j+1})$$

$F_x = F_{i,j} - F_{i-1,j}$, in point $(i - \frac{1}{2}, j)$;

$F_y = F_{i,j} - F_{i,j-1}$, in point $(i, j - \frac{1}{2})$;

$$\overline{\overline{F}}_{i+1/2,j+1/2} = \tfrac{1}{4}(F_{i,j} + F_{i,j+1} + F_{i+1,j} + F_{i+1,j+1})$$

$$\left\langle \frac{\partial u}{\partial x} \right\rangle_{i+\frac{1}{2},j} = \frac{1}{2\Delta s}(u_{i+\frac{3}{2},j} - u_{i-\frac{1}{2},j});$$

$$\left\langle \frac{\partial u}{\partial y} \right\rangle_{i+\frac{1}{2},j} = \frac{1}{2\Delta s}(u_{i+\frac{1}{2},j+1} - u_{i+\frac{1}{2},j-1});$$

$$\left\langle \frac{\partial v}{\partial x} \right\rangle_{i,j+\frac{1}{2}} = \frac{1}{2\Delta s}(v_{i+1,j+\frac{1}{2}} - v_{i-1,j+\frac{1}{2}});$$

$$\left\langle \frac{\partial v}{\partial y} \right\rangle_{i,j+\frac{1}{2}} = \frac{1}{2\Delta s}(v_{i,j+\frac{3}{2}} - v_{i,j-\frac{1}{2}});$$

$$\left\langle \frac{\partial^2 u}{\partial x^2} \right\rangle_{i+\frac{1}{2},j} = \frac{1}{\Delta s^2}\left(u_{i+\frac{3}{2},j} - 2u_{i+\frac{1}{2},j} + u_{i-\frac{1}{2},j}\right);$$

$$\left\langle \frac{\partial^2 u}{\partial y^2} \right\rangle_{i+\frac{1}{2},j} = \frac{1}{\Delta s^2}\left(u_{i+\frac{1}{2},j+1} - 2u_{i+\frac{1}{2},j} + u_{i+\frac{1}{2},j-1}\right);$$

$$\left\langle \frac{\partial^2 v}{\partial x^2} \right\rangle_{i,j+\frac{1}{2}} = \frac{1}{\Delta s^2}\left(v_{i+1,j+\frac{1}{2}} - 2u_{i,j+\frac{1}{2}} + u_{i-1,j+\frac{1}{2}}\right);$$

$$\left\langle \frac{\partial^2 v}{\partial y^2} \right\rangle_{i,j+\frac{1}{2}} = \frac{1}{\Delta s^2}\left(v_{i,j+\frac{3}{2}} - 2u_{i,j+\frac{1}{2}} + u_{i,j-\frac{1}{2}}\right)$$

When $k\Delta t \to (k + 1/2)\Delta t$, then

Implicit solution for ξ and *u*, and explicit solution for v to formula (1) in point (i, j)

$$\xi^{k+\frac{1}{2}} = \xi^{k} - \frac{\Delta t}{2\Delta s}\left(u^{k+\frac{1}{2}}(\overline{h}^{y} + \overline{\xi}^{x(k)})\right)_x - \frac{\Delta t}{2\Delta s}\left(v^{k}(\overline{h}^{x} + \overline{\xi}^{y(k)})\right)_y \qquad ...(4)$$

Implicit solution for ξ and *u*, and explicit solution for v to formula (2) in point $(i + \frac{1}{2}, j)$.

$$u^{(k+\frac{1}{2})} = u^{(k)} + \tfrac{1}{2}\Delta t f \overline{\overline{v}}^{(k)} - \tfrac{1}{2}\Delta t u^{(k+\frac{1}{2})}\left\langle \frac{\partial u^{(k)}}{\partial x} \right\rangle_{i+\frac{1}{2},j} - \tfrac{1}{2}\Delta t \overline{\overline{v}}^{(k)}\left\langle \frac{\partial u^{(k)}}{\partial y} \right\rangle_{i+\frac{1}{2},j}$$
$$- \tfrac{1}{2}\frac{\Delta t}{\Delta s} g \xi_x^{(k+\frac{1}{2})} - \tfrac{1}{2}\Delta t g u^{(k)} \frac{\sqrt{(u^{(k)})^2 + (\overline{\overline{v}}^{(k)})^2}}{(\overline{h}^{y} + \overline{\xi}^{x(k)})(\overline{c}^{x})^2}$$
$$+ \frac{\Delta t}{2} A_H \left(\left\langle \frac{\partial^2 u^{(k)}}{\partial x^2} \right\rangle_{i+\frac{1}{2},j} + \left\langle \frac{\partial^2 u^{(k)}}{\partial y^2} \right\rangle_{i+\frac{1}{2},j} \right) \qquad ...(5)$$

Implicit solution for ξ and *u*, and explicit solution for v to formula (3) in point $(i, j + \frac{1}{2})$.

$$v^{(k+\frac{1}{2})} = v^{(k)} - \tfrac{1}{2}\Delta t f \overline{\overline{u}}^{(k+\frac{1}{2})} - \tfrac{1}{2}\Delta t \overline{\overline{u}}^{(k+\frac{1}{2})}\left\langle \frac{\partial v^{(k)}}{\partial x} \right\rangle_{i,j+\frac{1}{2}} - \tfrac{1}{2}\Delta t v^{(k+\frac{1}{2})}\left\langle \frac{\partial v^{(k)}}{\partial y} \right\rangle_{i,j+\frac{1}{2}}$$
$$- \tfrac{1}{2}\frac{\Delta t}{\Delta s} g \xi_y^{(k)} - \tfrac{1}{2}\Delta t g v^{(k+\frac{1}{2})} \frac{\sqrt{(\overline{\overline{u}}^{(k+\frac{1}{2})})^2 + (v^{(k)})^2}}{(\overline{h}^{x} + \overline{\xi}^{y(k+\frac{1}{2})})(\overline{c}^{y})^2} +$$
$$\frac{A_H \Delta t}{2}\left(\left\langle \frac{\partial^2 v^{(k)}}{\partial x^2} \right\rangle_{i,j+\frac{1}{2}} + \left\langle \frac{\partial^2 v^{(k)}}{\partial y^2} \right\rangle_{i,j+\frac{1}{2}} \right) \qquad ...(6)$$

Similarly, the equation dispersion when $(k + 1/2)\Delta t \to (k + 1)\Delta t$ is as same as above.

Basic ecological control equations

$$\frac{\partial(HE)}{\partial t} + \frac{\partial(HuE)}{\partial x} + \frac{\partial(HvE)}{\partial y} = \frac{\partial}{\partial x}\left(H\lambda_x \frac{\partial E}{\partial x}\right) + \frac{\partial}{\partial y}\left(H\lambda_y \frac{\partial E}{\partial y}\right) + S_E \qquad ...(7)$$

$$\frac{\partial(HN)}{\partial t} + \frac{\partial(HuN)}{\partial x} + \frac{\partial(HvN)}{\partial y} = \frac{\partial}{\partial x}\left(H\lambda_x \frac{\partial N}{\partial x}\right) + \frac{\partial}{\partial y}\left(H\lambda_y \frac{\partial N}{\partial y}\right) + S_N \qquad ...(8)$$

where, $S_E = H\left[c_1(E_0 - E) - a\frac{E}{E_m + E}N\right]E$

$$S_N = H\left[-c_2 + b\frac{E}{E_m + E}\right]N$$

$$\lambda_x = 5.93\sqrt{g}h|u|c^{-1}, \ \lambda_y = 5.93\sqrt{g}h|v|c^{-1}$$

Where, E is marine nutrient; N is biomass density of red tide, λ_x is the eddy diffusion coefficient is in x direction; λ_y is the eddy diffusion coefficient is in y direction; ξ is added

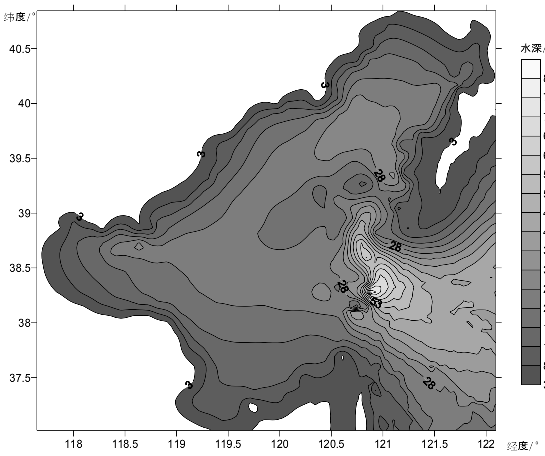

Fig. 1: Bathymetric chart of Bohai terrain.

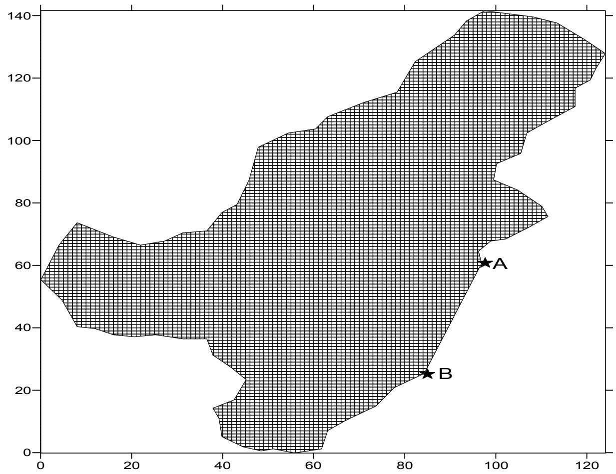

Fig. 2: Grid sketch of the Bohai Sea.

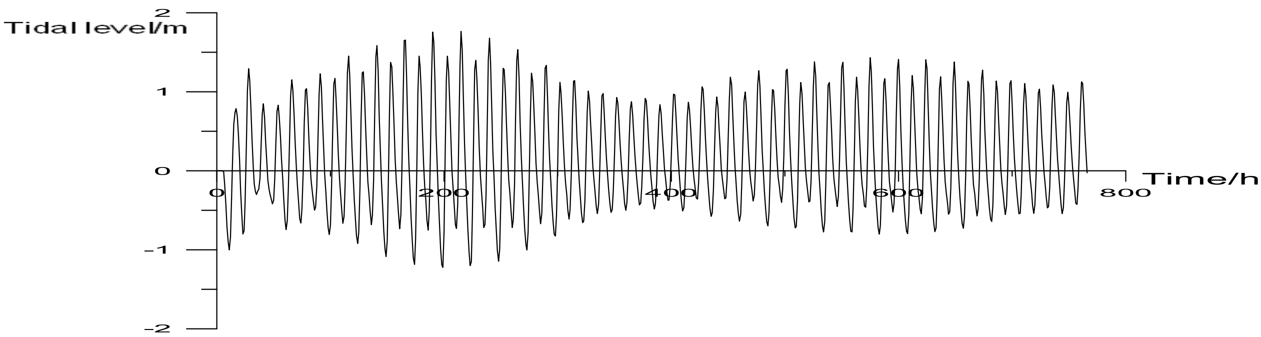

Fig. 3: Tidal level process of border point A.

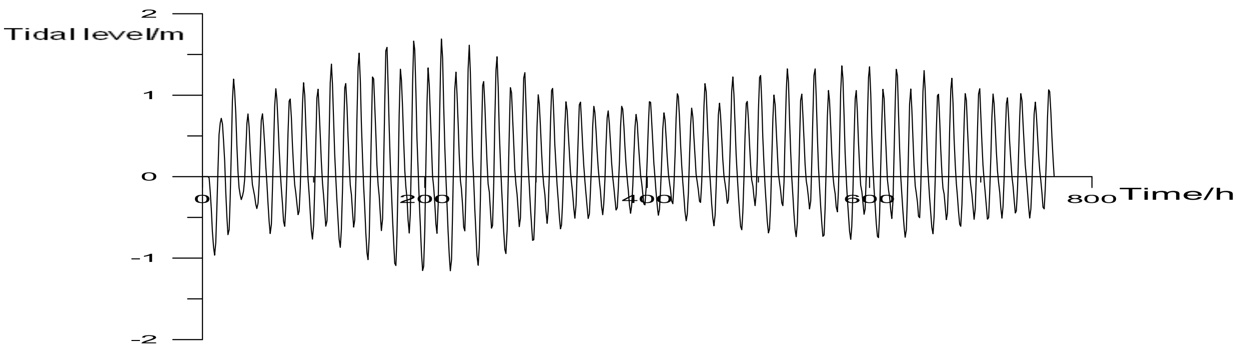

Fig. 4: Tidal level process of border point B.

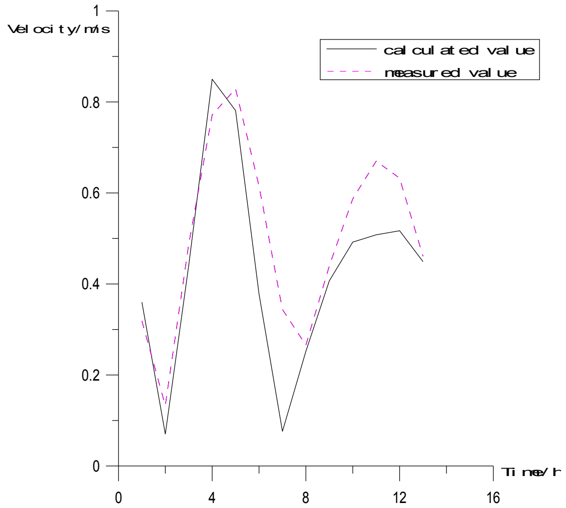

Fig. 5: Flow rate verification of measuring point.

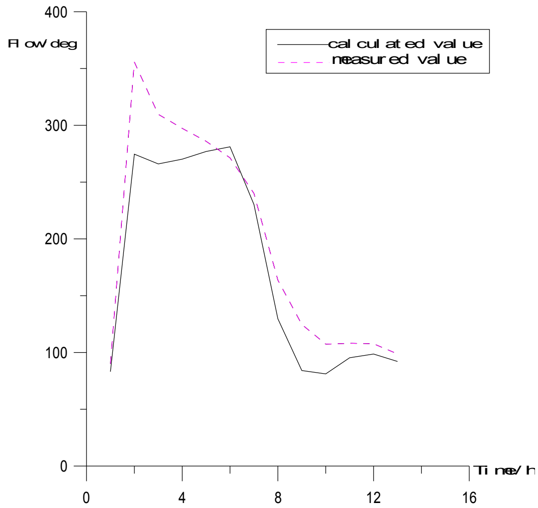

Fig. 6: Flow direction verification of measuring point.

Fig. 7: Remote sensing map of red tide on June11, 2004.

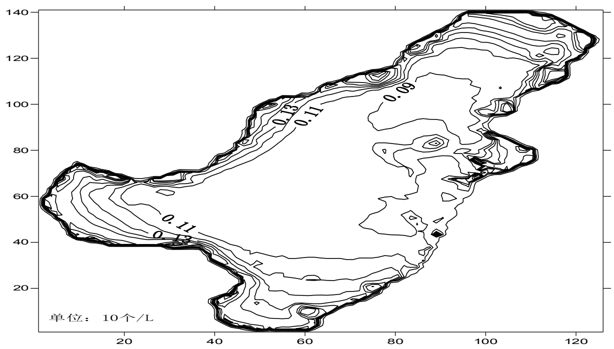

Fig. 8: Algal density distribution on June 6, 2004.

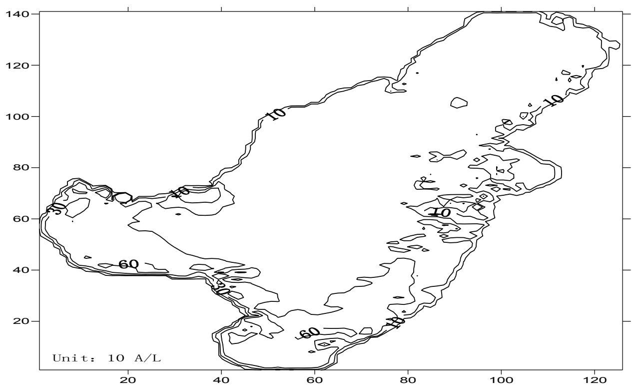

Fig. 9: Algal density distribution on June 11, 2004.

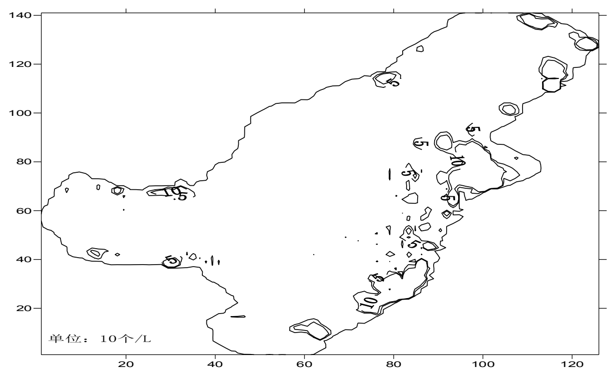

Fig. 10: Algal density distribution on June 16, 2004,

tide level; $H = h + \xi$, h is tide level; u is the average velocity in x direction; v is the average velocity in y direction; g is the gravitational acceleration; $c = \frac{1}{n} H^{\frac{1}{6}}$ is the Chezy coefficient, n is roughness: S_E and S_N are ecological items; c_1 and c_2 respectively mean the nutrient environment attrition rate and mortality rate of algae; a and b respectively mean the maximum absorption rate of algae and conversion rate of algae nutrition; E_0 and E_m respectively mean the nutrient input and nutrient-saturated constant.

Dispersion of upwind scheme finite difference: When, $k\Delta t \to (k+1)\Delta t$,dispersing to E for formula (7)in point (i, j):

$$
\begin{aligned}
E_{i,j}^{k+1} = & \frac{\Delta t}{H_{i,j}^{k+1}\Delta x^2}\left[(H\lambda_x)_{i-1,j}^k\left(E_{i-1,j}^k - E_{i,j}^k\right) + (H\lambda_x)_{i,j}^k\left(E_{i+1,j}^k - E_{i,j}^k\right)\right] \\
& + \frac{\Delta t}{H_{i,j}^{k+1}\Delta y^2}\left[(H\lambda_y)_{i,j-1}^k\left(E_{i,j-1}^k - E_{i,j}^k\right) + (H\lambda_y)_{i,j}^k\left(E_{i,j+1}^k - E_{i,j}^k\right)\right] \\
& + \frac{\Delta t}{H_{i,j}^{k+1}}\left[H_{i,j}^{k+1}\left(c_1\left(E_0 - E_{i,j}^k\right) - a\frac{E_{i,j}^k}{E_m + E_{i,j}^k}N_{i,j}^k\right)E_{i,j}^k\right] \\
& + \frac{(HE)_{i,j}^k}{H_{i,j}^{k+1}} - \frac{\Delta t}{\Delta x H_{i,j}^{k+1}}\left((HuE)_{i,j}^k - (HuE)_{i-1,j}^k\right) \\
& - \frac{\Delta t}{\Delta y H_{i,j}^{k+1}}\left((HvE)_{i,j}^k - (HvE)_{i,j-1}^k\right)
\end{aligned}
\quad ...(9)
$$

Dispersing to N for formula (8) in point (i, j):

$$
\begin{aligned}
N_{i,j}^{k+1} = & \frac{\Delta t}{H_{i,j}^{k+1}\Delta x^2}\left[(H\lambda_x)_{i-1,j}^k\left(N_{i-1,j}^k - N_{i,j}^k\right) + (H\lambda_x)_{i,j}^k\left(N_{i+1,j}^k - N_{i,j}^k\right)\right] \\
& + \frac{\Delta t}{H_{i,j}^{k+1}\Delta y^2}\left[(H\lambda_y)_{i,j-1}^k\left(N_{i,j-1}^k - N_{i,j}^k\right) + (H\lambda_y)_{i,j}^k\left(N_{i,j+1}^k - N_{i,j}^k\right)\right] \\
& + \frac{\Delta t}{H_{i,j}^{k+1}}\left[H_{i,j}^k\left(-c_2 + b\frac{E_{i,j}^k}{E_m + E_{i,j}^k}\right)N_{i,j}^k\right] \\
& + \frac{(HN)_{i,j}^k}{H_{i,j}^{k+1}} - \frac{\Delta t}{\Delta x H_{i,j}^{k+1}}\left((HuN)_{i,j}^k - (HuN)_{i-1,j}^k\right) \\
& - \frac{\Delta t}{\Delta y H_{i,j}^{k+1}}\left((HvN)_{i,j}^k - (HvN)_{i,j-1}^k\right)
\end{aligned}
\quad ...(10)
$$

RESULTS AND DISCUSSION

Bohai Sea is about 300 nautical miles north and south, 160 nautical miles east and west, the shape along the tube is like a gourd. The average depth of Bohai Sea is about 18 m, most areas of Bohai are shallow (Marine Geology 1985). Fig. 1 is the bathymetric chart of Bohai terrain. The model grid dividing is used by a square grid, the time step for the 60 s, space step for the 3 km, after the partition 127 × 142 for the trip. The grid sketch of model is shown in Fig. 2.

The two-dimensional ecological mathematical model of red tide is applied in Bohai Sea to simulate the actual *Phaeocystis* red tide which occurred on 11th-16th June, 2004. Where, the water boundary conditions are used as the tidal level process of border point A (27, 35) and B (27, 32) (shown in Fig. 2) from 7:00 May 30th to 23:00 June 30th 2004, as shown in Fig. 3 and Fig. 4. Then the model is verified by measuring points of the Bohai Sea, the velocity and flow verification sketch is shown in Fig. 5 and Fig. 6, and the verification result is good.

The scope of this *Phaeocystis* red tide is shown in Fig. 7, derived by the EOS / MODIS satellite remote sensing photos. The ecological model is applied to simulate the progress of this *Phaeocystis* red tide during one month, Fig. 8, Fig. 9 and Fig. 10 respectively show the distribution of algal density on June 6, June 11 and June 16 in 2004, which can reflect the growth progress of algae, birth and death. The area of the red tide derived from the EOS / MODIS satellite remote sensing photos is 2862 km^2, compared with that from the mathematical model 2637km^2, which shows the reliability of the red tide model, and the result is satisfactory.

CONCLUSION

In the paper, the researchers simulated the actual *Phaeocystis* red tide by the established two-dimensional ecological mathematical model. The simulation result coincides well with that from remote sensing photos, which means that the mathematical model can be applied for the red tide prediction.

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