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Application of Fuzzy Mathematics for Assessment of Water Quality

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ABSTRACT

With the development of urbanization and the improvement of the people's living standards, the contradiction against water supply and water demand is becoming more and more serious. And the development of the economy led to serious pollution of water, so it is essential to understand the water quality. This paper discussed the choice of the evaluated factors, the operations of the weight value, and compares with the status of the water quality in course of the evaluation. The conclusions indicated that the result of the fuzzy mathematics evaluation represented the water quality of surface waters. The evaluation result was compared with that of the traditional method and the reported results. It is indicated that the performance of the proposed model is practically feasible in the application of water quality assessment and its application is simple.

INTRODUCTION

The significance of water quality assessment is to judge the possibility and necessity of the development of water resources and maintaining a balanced ecosystem according to the water quality. In the development and utilization of water resources, the main projects affected by water quality due to the practical value and economic benefits include: agricultural irrigation, drinking water, aquaculture, water sports, industrial water and tourism. It is very important to analyse and assess the water quality. At present, many scholars develop many methods to assess water quality, such as neural network, grey method and fuzzy method and so on. Among them, the fuzzy method is widely used. Yin Cui-Qin (2007) applied fuzzy mathematics assessment to the water quality of surface water in Huadu District of Guangzhou. Guan Yanhai (2008) applied fuzzy mathematics method for groundwater quality evaluation in Tianjin city. Liu Rong-Zhen (2007) use fuzzy evaluation method to water quality of Yangtze River.

Fuzzy mathematics was proposed by professor Zadeh in 1965, which is widely used in each domain. In water quality assessment, fuzzy mathematics has many research achievements. The application of fuzzy mathematics is successful in assessment of river water quality, groundwater quality and lake water quality. When assessing surface water quality, the value of water quality is between the two national normal values and it is difficult to judge which grade the water quality belongs to. So the fuzzy mathematics is usually used to evaluate the water quality.

FUZZY EVALUATION MODEL

Fuzzy comprehensive evaluation is a quantitative evaluation method. Firstly, the main factors affecting the water quality are analysed. According to the judgment matrix, water quality is evaluated by varying degrees. And comprehensive quantitative evaluation results can be obtained by fuzzy mathematics method, which can provide the management decision. Because there are advantages using this method, it has been widely used in engineering design, economic management, water quality evaluation, etc. The method and steps of the fuzzy evaluation can be described as follows.

Establishment of Evaluation Factors Set

Evaluating a thing, if there are *n* evaluation index factors, denoted as u1, u2, u3, ..., un, then the *n* evaluation factors constitute a finite set of evaluation factors U:U = (u1, u2, u3, …, ui, …, un).

Establishment of Evaluation Set

If the actual level is divided into *m* levels, denoted as v1, v2, ..., vm, then constitute a limited set of comment V:V = (v1, v2, ..., vj, ..., vm).

Fuzzy Comprehensive Evaluation Matrix

Constructing a single factor model. Factors such as a decision-making: scientific. After poll, it shows that 15% of people say "very good" and 45% said it was "good", 30% of people say, "General", 10% of people say "poor", then the result can be described by fuzzy sets R1.R1= [0.15, 0.45, 0.30, 0.10]. When the fuzzy mathematics is applied to assess water quality, the degree of membership is used to describe the fuzzy boundary. The membership of the single index of pollutants (x_i) to the water grade (c_i) is made up of a fuzzy relation matrix. Membership degree can be expressed by membership function. There are many methods to solve the function, such as value law and subordination function method according to the function distribution shape curve. In this paper, the second method is used.

Supposing n is the number of pollutant index, x_{ki} is the observed value of pollutant index i in the k^{th} water sample, $C_{ij}(i=1,2\cdots\cdots n;\ j=1,2\cdots\cdots m)$ is the standard of j grade. The membership function of pollutant index to all grades of water:

When $j=1$

$$y_n=\begin{cases}1 & x_{ki}\le c_{i1}\\ \dfrac{c_{i2}-x_{ki}}{c_{i2}-c_{i1}} & c_{i1}<x_{ki}\le c_{i2}\\ 0 & x_{ki}>c_{i2}\end{cases}$$

When $j = 2, 3, \ldots, n$-1,

$$y_n=\begin{cases}1-\dfrac{c_{ij}-x_{ki}}{c_{ij}-c_{ij-1}} & c_{ij-1}\le x_{ki}\le c_{ij}\\ \dfrac{c_{ij+1}-x_{ki}}{c_{ij+1}-c_{ij}} & c_{ij}<x_{ki}\le c_{ij+1}\\ 0 & c_{ij+1}<x_{ki}<c_{ij-1}\end{cases}$$

$j=m$

$$y_n=\begin{cases}0 & x_{ki}<c_{in-1}\\ 1-\dfrac{c_{ij}-x_{ki}}{c_{in}-c_{in-1}} & c_{in-1}\le x_{ki}\le c_{in}\\ 1 & x_{ki}>c_{in}\end{cases}$$

If x_{ki} is the known value, the membership of the i^{th} pollutant index in k^{th} water sample to all kinds of grade of water. The fuzzy matrix R can be obtained as follows.

$$R=\begin{bmatrix}y_{11}\,y_{12} & \bullet & \bullet & \bullet & y_{1j}\\ y_{21}\,y_{22} & \bullet & \bullet & \bullet & y_{2j}\\ \bullet\ \bullet & & \bullet & & \bullet\\ \bullet\ \bullet & & \bullet & & \bullet\\ \bullet\ \bullet & & \bullet & & \bullet\\ y_{i1}\,y_{i2} & \bullet & \bullet & \bullet & y_{ij}\end{bmatrix}$$

Determination of the Index Weights

In the actual evaluation, the importance of evaluation of each factor has been often different. Taking into account the objective reality, the collection is actually a factor in the evaluation factors set U, a fuzzy set A on the domain; it is a finite set.

$W = w_1/u_1 + w_2/u_2 + \ldots + w_n/u_n$

It can also be an n-dimensional vector

$W = (w_1, w_2, \ldots, w_i, \ldots, w_n)$

Its domain is U, w_i is the membership degree of the corresponding element, where $wi\in[0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Determination of the Grade Weighting Matrix

Rank weighting matrix is to assign without considering vague boundary conditions. It reflects the influencing degree of each evaluation grade on the final result, and its value can also be a series of specific numbers, but also for the value of each sub-interval, the general percentile, in accordance with the corresponding factors to form a weighting matrix.

$C = (c_1, c_2, \ldots c_j \ldots c_m)\ (0 \le c_j \le 100)$

Comprehensive Evaluation

$$B=R\cdot C^T=\begin{bmatrix}r_{11} & \cdots & r_{1m}\\ \vdots & r_{ij} & \vdots\\ r_{n1} & \cdots & r_{nm}\end{bmatrix}\cdot(c_1, c2, \cdots c_m)^T=\begin{bmatrix}b_1\\ \vdots\\ b_i\\ \vdots\\ b_n\end{bmatrix}$$

Table 1: Measured values of the monitoring section.

Section	Concentration (mg/L)						
	DO	BOD_5	COD	Cyanide	Volatile Phenols	Ammonia	pH
I	7.0	3.0	15	0.04	0.003	0.4	7
II	6.5	3.3	17	0.045	0.004	0.6	7
III	5.0	3.6	22	0.055	0.006	1.2	6.8
IV	4.0	5.0	29	0.050	0.005	0.7	7.8
V	5.5	3.7	17	0.044	0.004	0.8	7.5

Table 2: Surface water quality standards" (GB3838-2002).

Factors	Level				
	I	II	III	IV	V
DO	7.5	6	5	3	2
BOD_5	2	3	4	6	10
COD	10	15	20	30	40
Cyanide	0.005	0.05	0.1	0.15	0.2
Volatile Phenols	0.001	0.002	0.005	0.01	0.1
Ammonia	0.15	0.5	1.0	1.5	2.0

Index value (mg/L)

Where, $0 \le b_i \le 100$

According to Index score of various factors, you can come to a final composite score evaluation object value M.

$$M = W \cdot B = (w_1 w_2 \cdots w_i \cdots w_n) \cdot \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix} \quad (0 \le M \le 100)$$

APPLICATION

The fuzzy mathematic model is used to evaluate the water quality which can be seen in Table 1. The process is as follows.

Establishment of the Factors Set

According to the characteristics of the river and water monitoring data, five main factors are selected.

$$u = \{Do_i, BOD, COD, (CN)2_i, C_6H_5OH_i, NH3 - NH4_i\}$$

Establishment of Evaluation Set

According to "surface water quality standards" (GB3838-2002), the evaluation level is divided into five levels, V = {I, II, III, IV, V}.

Determination of the Membership Functions and Membership Degree

To do as an example, measurement values are between level I and level IV. According to the formula, its membership is:

$$\mu_{vi}(u_j) = \begin{cases} (u_j - a_i)/(c_j - a_j), a_j \le u_j \le c_j \\ (b_j - u_i)/(b_j - c_j), c_j \le u_j \le b_j \\ 0, \quad else \end{cases}$$

$$R1 = \begin{bmatrix} 0.3333 & 0.6667 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ 0.2222 & 0.7778 & 0 & 0 & 0 \\ 0 & 0.6667 & 0.3333 & 0 & 0 \\ 0.2857 & 0.7134 & 0 & 0 & 0 \end{bmatrix}$$

$$R2 = \begin{bmatrix} 0.6667 & 0.3333 & 0 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0.1111 & 0.8889 & 0 & 0 & 0 \\ 0 & 0.3333 & 0.6667 & 0 & 0 \\ 0 & 0.8000 & 0.20 & 0 & 0 \end{bmatrix}$$

$$R3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 \\ 0 & 0.995 & 0.005 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 \end{bmatrix}$$

$$R4 = \begin{bmatrix} 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 & 0 \end{bmatrix}$$

$$R5 = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0.133 & 0.8667 & 0 & 0 & 0 \\ 0 & 0.3333 & 0.667 & 0 & 0 \\ 0 & 0.4000 & 0.6 & 0 & 0 \end{bmatrix}$$

Fuzzy Matrix Composite Operator

Fuzzy comprehensive evaluation is made in section IV and V, and compared between Model 1 and Model 2.

$$B4 = A4 \cdot R4 = (0 \quad 0.2007 \quad 0.3413 \quad 0.4580 \quad 0)$$

B5=A5×R5= (0.0144 0.4950 0.4906 0 0)

Seen from the above, the maximum value of B is 0.4580, according to the maximum principle of membership, water quality assessment rating is level IV in section IV, while in section V, water quality assessment is level II.

Compared with the single index method, there are some differences in the results in section IV and V. Section V fuzzy comprehensive evaluation method when the water reached the stage II. When the single factor evaluation III, the water reached the level of Class II. Anyway, water calculated using the fuzzy comprehensive evaluation method is better than the single factor evaluation of water quality obtained.

CONCLUSIONS

Application of fuzzy mathematics on the water quality evaluation is a novel research area. Based on the modeling results obtained in this study, the following conclusions can be drawn:

1. The fuzzy method is proposed by virtue of the dynamic characteristics of water quality, which increased the evaluation precision. Therefore, the method is reliable and effective.
2. The fuzzy method which can overcome the disadvantages of single factors can reflect the water quality at present. This model can approach the reality.
3. This method is not only suitable for the evaluation of the river water quality, but also suitable for other aspects such as groundwater quality, prediction of quality in the atmospheric environment, and so on.

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