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Original Research Paper

Comparative Study on the Modelling of Mixing Length Distribution in the Sediment-Laden Flow

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ABSTRACT

In view of the established mixing length model with lack of outside China applicability, different forms of flow must be adopted. Firstly, from motion mechanism of mixing-length of the sediment-laden flow, a new mixing-length distribution mode in vertical was deduced by combining flow turbulence theory with applied mathematics theory. Then, the existing typical experimental data were verified well, and by comparing linear distribution and parabolic distribution, the result shows that the distribution is superior to the latter two. Finally, its dynamics lead to mixing-length distribution characteristic is also discussed preliminarily, and influence of the sediment concentration on the distribution of the mixing-length. The established expression on the vertical distribution of the mixing-length is not only simple and explicit, but also better reflects the motion characteristics of the complex flow, for the foundation of further study on the velocity profile of the sediment flow.

INTRODUCTION

The mixing length *l* is an important dynamic characteristic of turbulent flow, and its vertical distribution is closely related to the water resistance, sediment movement and environmental problems. It is directly impacted on the evolving process of bed sediment erosion and deposition. Therefore, it has the important theoretical significance to study the vertical distribution of the mixing-length in the sediment-carrying flow, and it also helps solving some practical engineering problems. For these reasons, many home and abroad scholars have done a large number of experimental measurements and analysis; most of these studies were undertaken by the water test in the open channel, pipe and plate boundary layer.

The concept of mixing length was introduced to refer to the concept of free trip of molecular collisions by Prandtl (1925), which says that when the layer of fluid particles due to the pulse from the speed jump to another level, the fluid particle just convert the original momentum and is equal to the momentum of the new layer. The distance between the two layers is called mixing length *l*. According to the mixing length theory of Prandtl, in the near-wall turbulent flow, it is assumed that *l* is proportional to the normal distance *y* from the solid wall. That is:

$$l = k\,y \quad ...(1)$$

Where *l* is the mixing length, it is a variable and related to the movement of the flow, and it can be determined by the test; κ is the Karman constant, and by the experiment, it can be determined that $k \approx 0.4$.

For the fully developed pipe flow and open channel flow, Nikuradse (1957) proposed the following formula for the distribution of the mixing length:

$$\frac{l}{R} = 0.14 - 0.08(1 - y/R)^2 - 0.06(1 - y/R)^4 \quad ...(2)$$

Where *R* is the radius of the pipe or the open channel of water depth.

Near the region of the wall, the results on (*1*) are consistent with Nikuradse's. However, near the absent region of the wall, viscosity plays a decisive role. We usually use Van Driest (1956) formula to correct the length of the linear relationship of mixing length:

$$l = ky\left[1 - \exp\left(-\frac{y(t_w / r)^{1/2}}{Av}\right)\right] \quad ...(3)$$

Where $A = 26$ and τ_ω is the shear stress of the wall.

Avoiding confirming *l*, von Karman had suggested that establishing the contact of distribution of the *l* and the average velocity. That is:

$$l = k\left|\frac{\partial U / \partial y}{\partial^2 U / \partial y^2}\right| \quad ...(4)$$

For this formula, we can get better results with the type of near-wall flow, but, it lacks the versatility. For example, when in jet and wake, and the velocity profile with inflection point, using eq. (4) we will come to an infinite mixing length. Therefore, von Karman's formula is rarely used.

Other scholars through the use of the formula on the velocity gradient deduced the mixing length along the entire depth of sediment-laden flow, ignoring the viscous shear stress in the mainstream area, that is:

$$t = r l^2 \left(\frac{du}{dy} \right)^2 \qquad ...(5)$$

Based on the mainstream area, the turbulence shear stress with sediment-laden flow is essentially linear distribution:

$$t = t_b \left(1 - \frac{y}{D} \right) \qquad ...(6)$$

Where $t_b = r\left|u_*\right|u_*$ is the bed resistance, r is the water density, u_* is the bed friction velocity, y is the vertical coordinate and D is water depth.

Uniting eq. (5) and eq. (6), we get:

$$l = ky\sqrt{1 - y/D} \qquad ...(7)$$

Where k is the Karman constant, and by the experiment, it can be determined that $k = 0.4$.

Thus, the mixing length model is lacking abroad applicability, but if assuming different mixing lengths, then we will get different flow distributions. For the simple type of shear layer flow, different forms of flow can use different empirical constants. But for the complex flow that plays an important role in the turbulent transport processes, it is difficult to determine the *l*. So, in this paper, basing on the movement mechanism of the mixing length of sediment-laden flow, the vertical distribution of mixing length mode of sediment-laden is established from the perspective of fractal scale and its coefficients are determined using the least square method and the corresponding boundary conditions. With verifying the field and test data, and comparing with other formulas, the formula which was deduced in this paper is not only simple and explicit, but also better reflects the vertical distribution characteristics of the mixing length of sediment-laden flow.

DERIVATION OF EQUATION

Assuming the mixing length defined as follows: Setting up a fluid block group, it moves with its original time-averaged velocities. Because of turbulence, the block group moves from y_1 to y_2 in the lateral. At point y_2, the difference between its original velocity and velocity-average is *DU.* If the average transverse velocity of point y_2 is just *DU* then the distance of $y_2 - y_1$ is defined as the mixing length *l*. As sediment concentration distribution in sediment-laden flow is not uniform, and with flow turbulence effects and other factors, "whirlpool" water mass movement in the sediment-laden flow is actually a random motion. The relationship between the actual length *l* of the trajectory of water masses with the length of horizontal projection can be described by fractal theory. Considering Fig. 1, it shows that the fractal structure, when fractal iteration *i* is 0, it is a straight line; when *i* is *1*, the line segment between 0 to l_1 remain unchanged, introducing any at point l_3 in the segment between l_1 and l_2 above (or below), after 1 iteration, the linear variation is from 0-l_1-l_2 to 0-l_1-l_3-l_2. In accordance with such rules of fractal iteration such as Fig. 1 (c) and Fig. 1 (d), *i-1* iterations of the fractal shape and *i* iterations of the fractal shape are represented by the dotted line and solid line respectively.

The average relationship between the length of line after fractal iteration *i* times and the level mapping distance is,

$$l_i = b^{-i} y_i \qquad ...(8)$$

Where l_i is the actual length of the line through the fractal iteration; y_i correspond to the projection of its length, the coefficient β is parameter in order to reflect the average relationship between the actual length of the line and the average length of the level mapping.

According to fractal theory, fractal iteration for any length of line *i* with the number of *y1 can* be expressed as:

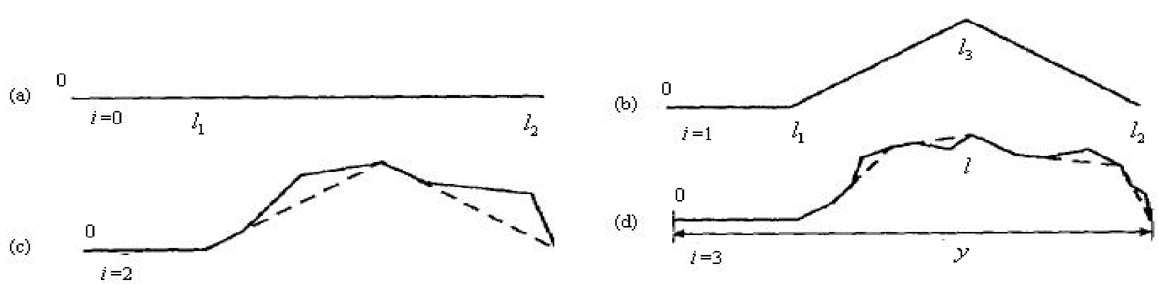

Fig.1: Diagram of water masses in the actual motion path length and level mapping distance.

$$N(y_i) = y_i^{-D} \quad ...(9)$$

Where D is the fractal dimension, for the one-dimensional case. $0 < D < 1$.

From eq. (8) and eq. (9), it is easy to know that the average relationship between actual water mass trajectory length l and the length y of the level mapping can be described as the following relationship:

$$l = N(y_i) \cdot l_i = b^{-i} y_i^{1-D} = A \cdot y^{1-D} \quad ...(10)$$

Where A is a proportionality constant, l is the mixing length and y is the water depth. Eq. (10) is the mixing length index formula of sediment-laden flow in this paper.

METHODS

Theoretically, eq. (10) should meet the measured value at each layer, however, for the observation error and the ignorance of the nonlinear interaction between them, then, there exists some error between the theoretical value and the measured value at each layer. Because of the number of variables is 2 and generally the measured data are not 2 layers, so the least square method combined with enumeration was used in this paper for solving. Concrete steps are as follows:

1. The establishment of objective function is:

$$f = \sum_{i=1}^{n} [l(y_i) - l_i]^2 = \sum_{i=1}^{n} \left\{ A y_i^{1-D} - l_i \right\}^2 \quad ...(11)$$

Where y_i and l_i respectively represent the water depth and corresponding mixing length near the bottom of the *i*-th layer; n is the total number of the layers.

2. Enumeration D, ranging from 0.0001 to 1m, with an increment of 0.0001, which results in 10000 groups totally.

3. For each D, if the objective function reaches the minimum, the following conditions are satisfied.

$$\frac{\partial f}{\partial A} = 0 \Rightarrow A \sum_{i=1}^{n} y_i^{2(1-D)} = l_i \sum_{i=1}^{n} y_i^{1-D} \quad ...(12)$$

Where n is the total number of the layers for fitting. Solving eq. (7) with the matrix method, then gives A.

4. Choosing the minimum of objective function f from the 10000 groups, we can get the value of D.

VERIFICATION AND COMPARISON

In order to approve the reasonableness of eq. (10), the formula in this paper was verified by adopting the existing typical experimental data, and making a detailed comparison with the results of eq. (1) and eq. (7).

The experimental value, the formula (1) fitting value, the formula (7) fitting value and the model and the measured values of the best fitting value were represented by real point, fine lines, dashed lines and solid lines respectively, and similarly used hereinafter.

From Figs. 2, 3, 4 and 5 it can be seen that the calculated value of this index eq. (10) is more close to the measured values than the eq. (1) and eq. (7), indicating that the former is more accurate and better reflect the vertical distribution of mixing length in the sediment laden flow.

PRELIMINARY ANALYSIS

Over the years, it often assumes a simple empirical formula to determine the mixing length l. For the free layer, l can be assumed to remain constant in the cross-section, and is proportional to the local thickness δ, the scale factor depends on the type of free flow. Thus, the mixing length model is the lack of versatility, for different forms of water must adopt different empirical constants. This is the inevitable result due to the mixing length theory ignores the turbulent diffusive transport and convective transport of momentum. This paper is about the movement mechanism of the mixing length in sediment-laden flow, combined with flow turbulence theory and application of mathematical theory, which deduced a new vertical distribution model of the mixing length. As can be seen from Table 1, eq. (10) calculated the fitting parameters A and D, its range of value is $0 < D < 1$. This is basically fitted with the range of the fractal dimension value D based on the fractal theory.

The results are shown on data of Einstein & Chien (1955) in group III from Table 1. It can be seen that eq. (10) degenerate to the Prandtl formula when experimental conditions are clear-water state shown C-13 group, $D = 0.0175$, $1 - D \approx 1$, A = 0.4026, then,

$$l = ky$$

Where k is the Karman constant, experiments show that $k = 0.40 \sim 0.41$.

In addition, we can view the near-wall region with a large concentration gradient as a stratified fluid. In the atmospheric boundary layer temperature stratification fluid, Liaxtman assumed mixing length

$$l_m = B y^{1-e} \quad ...(13)$$

Where parameters B, e are related to its stratification and roughness.

Comparing eq. (10) and eq. (13), although there are different theories that have different perspectives in the causation of stratification fluid between the atmospheric boundary layer and the sediment-laden flow, stratification

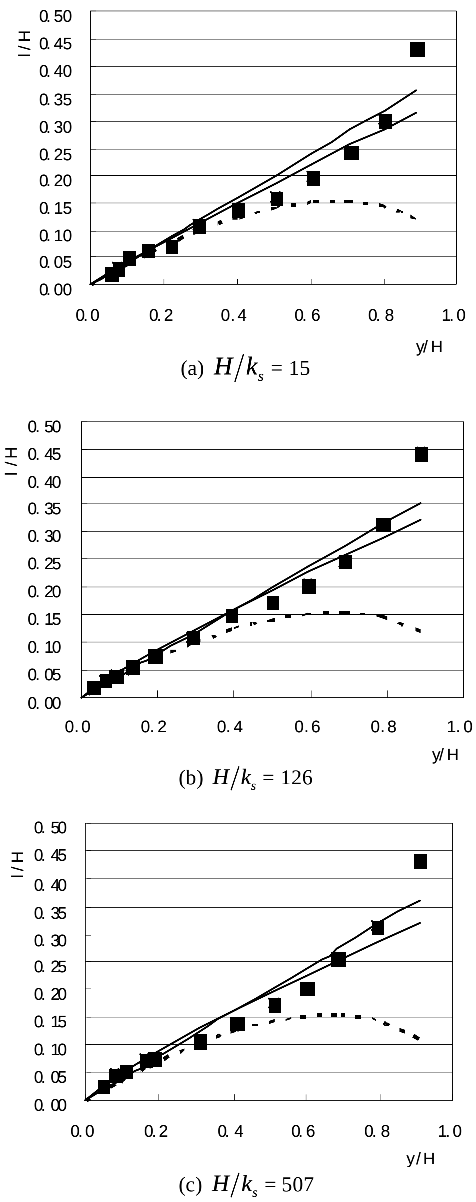

Fig. 2: Validation and comparison of experimental data with Nikuradse (1933).

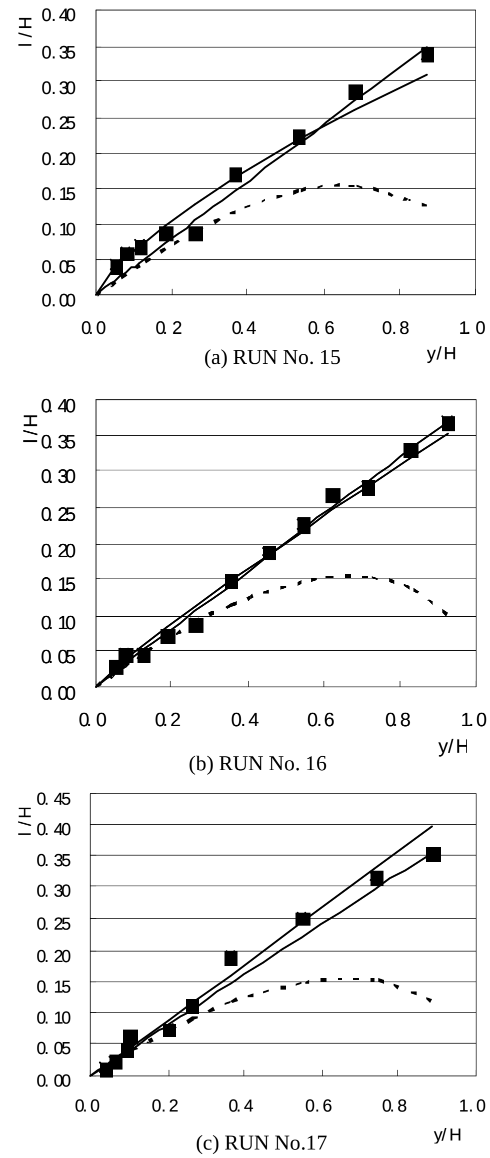

Fig. 3: Validation and comparison of experimental data with Vanoni (1946).

of sediment-laden flow is determined by the concentration gradient, which we call density stratification. Stratification of atmospheric boundary layer is determined by temperature and the distribution of pressure. However, both of which are stratification fluid, movement characteristics are determined by the stratification of the fluid should be similar. In the atmospheric boundary layer under the condition of non neutral equilibrium, these effects of fluid layer in turbulent exchange and concentration stratification in sediment-laden flow on the turbulent exchange that are similar, and the type has same expression. Therefore, this also indicates that the mixing length formula in sediment-laden flow established has a certain rationality.

In order to analyse sediment concentration affected the mixing length, this paper extracted the measured sediment (Table 2) from the experimental data by Einstein & Chien (1955).

From Table 2, we can see that, when the parameter D is closer to 0, the less sediment concentration is in the flow. There is positive relationship between parameters D and sediment concentration, the greater the parameter D, the more the sediment concentration, and vice versa. Therefore, from

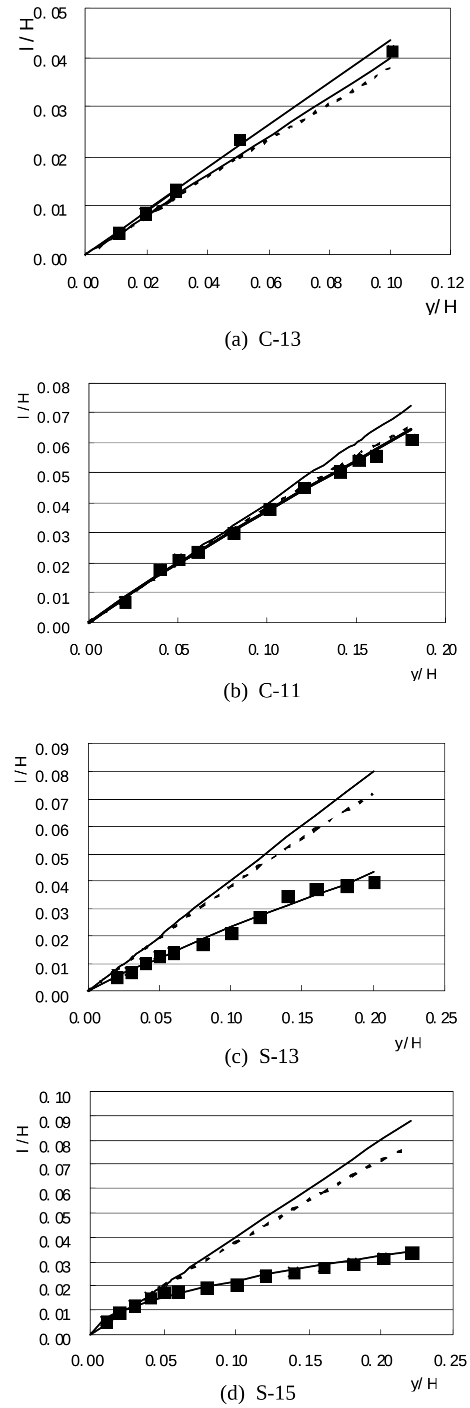

Fig. 4: Validation and comparison of experimental data with Einstein & Epik (1996).

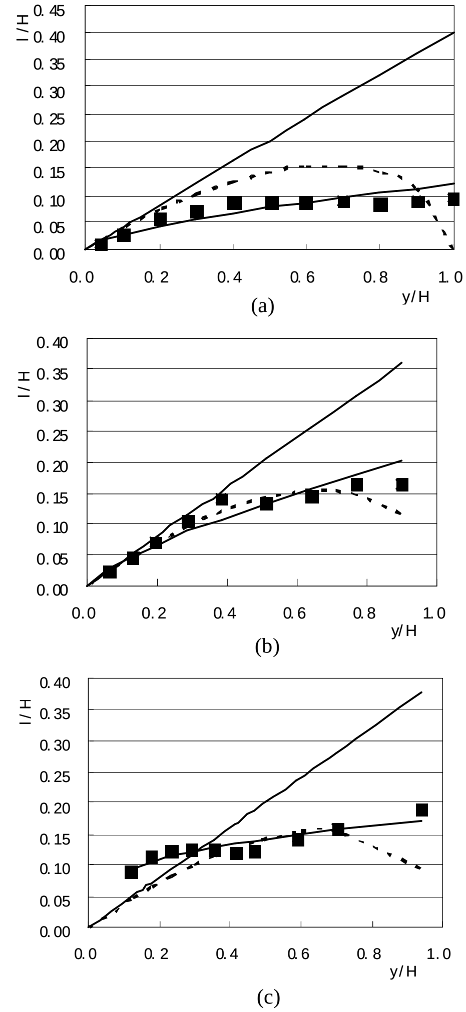

Fig. 5 : Validation and comparison of experimental data with Castro & Epik (1996).

combination of the eq. (10), it can be concluded that the sediment concentration is inversely proportional to the mixing length, that is, the greater the sediment, the smaller the mixing length.

The mixing length on vertical location is actually the location of different possible scale size-average of turbulent vortex. Turbulent water as a carrier, almost all the scales are involved in the vortex momentum transfer, mixing length is the average measurement of all swirls. Increasing the viscosity of sediment-laden flow will cause the disappearance of small-scale vortex.

In fact, the mixing length is also related to sediment carried by the vortex, the performance of the macro is associated with the sediment concentration. As the concentration increases, the constraints of the role of sediment on turbulence became increasingly obvious but the mixing length decreases with increasing concentration. In addition, the sediment concentration distribution is usually great on the small, concentration gradients to further restrain mixing effect. This inhibition may be a more obvious influence on the sediment

Table1: Each set of experimental data parameter values of D and A.

Experiments	Sets	D	A
Nikuradse (1933)	(a)	0.0658	0.3541
	(b)	0.1191	0.3587
	(c)	0.1625	0.3459
Vanoni (1946)	(a)	0.2655	0.3437
	(b)	0.0891	0.3780
	(c)	0.0246	0.4473
Einstein & Chien (1955)	(a) C-13	0.0175	0.4026
	(b) S-11	0.0798	0.2637
	(c) S-13	0.0903	0.1541
	(d) S-15	0.4573	0.0297
Castro & Epik (1996)	(a)	0.3451	0.1203
	(b)	0.2643	0.2173
	(c)	0.7115	0.1735

Table 2: Comparison of the parameter D with sediment from eq. (10).

Number	Parameter D	The largest vertical sediment concentration (g/L)
C-13	0.0175	0
S-11	0.0798	31
S-13	0.0903	252
S-15	0.4573	625

itself, causing further reduction of the mixing length. When the sediment concentration is large enough, the gradient inhibitory effect is obvious, mixing length will be less than mixing length of the water body (from Table 1 can also draw a comparison). Fine particles are more likely to exhibit phenomenon that the mixing length is less than the mixing length of the water body.

CONCLUSIONS

Based on the movement mechanism of the mixing length of sediment-laden flow, the vertical distribution model of mixing length of sediment-laden is established from the perspective of fractal scale and its coefficients are determined using the least square method and the corresponding boundary conditions in this paper. Through verification with field experimental data, and comparison with other calculated results, the following conclusions can be made.

1. The index formula is closer to the measured value and higher accuracy than other formula and the linear formula. It shows that the formula can reflect the vertical distribution of the mixing length of sediment-laden flow.
2 The research results show that this expression is explicit, simple in principle, reasonable in process, credible in result, and the establishment of the index formula of the mixing length of sediment laden flow has a certain rationality.
3. The greater the sediment, the smaller is the mixing length. At the same time, it also has a strong data basis and research value for further study on the velocity vertical distribution of sediment-laden flow.

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