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Original Research Paper

Water Environment Fuzzy Comprehensive Evaluation Based on Improved Set Pair Analysis (SPA)

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ABSTRACT

Set pair analysis (SPA) is a method for the application on analysis of risks and project decision evaluation. This method combines qualitative analysis and quantitative analysis. The previous SPA study only focused on three ranks evaluation. This paper presents an improved SPA method utilizing original relation degree malleability of SPA, and extends it to five ranks evaluation. The improved method can depict identity, discrepancy and contrary in more detail. In addition, this study combines fuzzy comprehensive evaluation method to evaluate four main factors of surface water environment, instead of the discussion of i^+, i^-, j^+, j^- coefficient. It not only obtains evaluation result but also obtains probability distribution of each rank. In this paper, we apply the method to factual example and compare the results with synthesis index method, and gray associated analysis method. We conclude that the improved method can represent the true state of the object with extensive engineering application value.

INTRODUCTION

Comprehensive evaluation is an all-round evaluation to systems on the technology, economy and society, in engineering, fuzzy factors is very frequent, especially on new production development and project pre-research. No matter personnel, environment and technology, it is in the existence of uncertain factors in every respect. At present, expert's marks, gray-relation method and fuzzy comprehensive evaluation method are applied widely, these methods may all comprehensive evaluate to uncertain factor systems, things and phenomenon. These methods have evaluation results generally: the worst, worse, common, better, the best, etc. Being some difficulty on defining membership grade, also being conflict between qualitative analysis and quantitative analysis, but fuzzy comprehensive evaluation based on improved SPA can solve these conflicts (Deng 2006).

SPA METHOD THEORY

Set pair analysis (SPA) is a new system analysis method, introduced by Zhao ke-qin in 1989 (He 2006).The method applies to the study of uncertainty from three aspects: identity, discrepancy, contrary. The method all-aroundly depicted the relations of two objects. SPA is an essentially new uncertainty theory, its core idea is that certainty and uncertainty of the object are regarded as a system of certainty and uncertainty; in this system, certainty and uncertainty are interrelated, mutual effect, mutual cancel, and they may mutual transform under certain conditions. The method depict fuzzy, random, information-fragmentary uncertainty by relation degree u, thereby transform uncertainty analysis into mathematics operation. Identity, discrepancy and contrary relation degree express are given as follows:

Hypothesis set $A_0 = [x_{0,1},\text{---}, x_{0k},\text{---}, x_{0,n}]$, set $B_1 = [x_{1,1},\text{---}, x_{1k},\text{---}, x_{1,n}]$, so A_0, B_1 compose set pair $H = (A_0, B_1)$, following analysis the set pair. A_0, B_1 contain corresponding n factors, s factors have little difference between set A_0 and set B_1 on quantity, p factors have great difference between set and set on quantity, the rest $f = n - s - p$ factors exist some difference on quantity, but the difference is not very obvious. If little difference is regarded as identity, great difference is regarded as contrary, some difference is regarded as discrepancy, so relation of set pair $H = (A_0, B_1)$, translates into relation of identity, discrepancy and contrary.

Commonly define: s/n is identity degree of set A_0 and B_1, token is a; f/n is discrepancy degree of set A_0 and B_1, token is b; p/n is contrary degree of set A_0 and B_1, token is c. In order to all-roundly depict total relation state of the two sets A_0 and B_1 by relation degree showing as:

$$u = (A_0, B_1) = \frac{s}{n} + \frac{f}{n} i + \frac{p}{n} j = a + bi + cj \qquad \text{...(1)}$$

In formula: a, b, c fulfil number normalization, namely $a + b + c = 1$; relation degree commonly express as above equation right formula; especially it is a numerical value, namely relation coefficient; i is discrepancy uncertainty coefficient, the range of the value is [-1,1] when i is -1 or 1,the system is certain, when change between -1 and 1, along with

$i \to 0$, uncertainty increase obviously, j is contrary coefficient, setting its value is -1 forever. The description is quantitative description to certainty and uncertainty, there into, *a, c* is certain relatively, and that *b* is uncertain relatively (Jiang 2007).

Core content of set pair analysis theory is certain and uncertain system, the system can be depicted by relation degree, but presently set pair analysis method needs improvement on the evaluation (Li 2000), as *b, c* parameter detail problem, namely identity, discrepancy and contrary remark detail problem. For example, quality evaluation of surface water contains five grades water quality criterion; remark of one scheme index contains: good, common, bad, in addition to contain: better, the best, worse, the worst, etc. So we need divide these indexes in more detail. Though, some scholars deal with these problems with approach degree (Liu 1997, Wang 2004), but it is not very perfect.

IMPROVED SPA EVALUATION MODEL

Set pair analysis method apply on comprehensive evaluation of groundwater environment, hypothesis *n* evaluation index, each grade criterion of groundwater environment index is $A_p = [v_{p1},---,v_{pk},---,v_{pn}]$, respectively composing a set, *p* express the grade, each region index numerical value of groundwater environment is $B_q = [q_{q1},---, v_{qk},---, v_{qn}]$, composing another set, *q* express the region, the two sets form a set pair (A_p, B_q). Comparing corresponding factors of the two sets x_{qk} and v_{pk}, if x_{qk} is in evaluation grade of v_{pk}, we may regard as identity, if x_{qk} is in adjacency grade of v_{pk}, and on superiority side of evaluation grade, token of its value is b^+; if x_{qk} is in adjacency grade of v_{pk} and on inferior side of evaluation grade, token of its value is b^-; if x_{qk} is in separated grade of v_{pk}, and on superiority side of evaluation grade, token of its value is c^+; if x_{qk} is in separated grade of v_{pk}, and on inferior side of evaluation grade, token of its value is c^-. In this way *b, c* can be more detailed, and the disposal accord with malleability of original relation degree. Formula (1) can transform into formula (2) or formula (3):

$$u=(A_0,B_1)=a+bi+cj=a+(b^+ +b^-)i+(c^+ +c^-)j \quad ...(2)$$

Or

$$u=(A_0,B_1)=a+bi+cj=a+(b_1i^+ +b_2i^-)+(c_1j^+ +c_2j^-) \quad ...(3)$$

St. $a+b^+ +b^- +c^+ +c^- =1$

$i^+ \in [0,1] \quad i^- \in [-1,0]$

And $i^+ + i^- = i \in [-1,1] \quad j^+ = \{0,1\},\ j^- = -1$

More analysis quantity relation between index grade criterion of groundwater environment and fact measure value of evaluation index, we can draw a conclusion, because of numerical discrepancy of evaluation index, even though groundwater environment index of different regions is in the same grade, sustainable-utilization of groundwater is different. Thereby, it is necessary to calculate relation degree of identity, discrepancy and contrary accurately. This research adopt approach degree method to ensure value of relation degree coefficient a, b, b_1, c_1, c_2. If index value is in the evaluation grade, following *a*-1 other coefficient is zero. If index value is in adjacency grade, the value of *a* increases more and more when approaching value of evaluation criterion, contrarily b_1, b_2, c_1, c_2 increase more and more; and on superiority side of evaluation grade, when approaching adjacency evaluation criterion *a* more and more big b_1, more and more small, contrarily *a*, more and more small, b_1 more and more big. If index value is in adjacency grade, and on inferior side of evaluation grade, when approaching adjacency evaluation criterion, *a* more and more big b_2, more and more small, contrarily, *a* more and more small, b_2 more and more big. If index value is in separated grade, and on superiority side of evaluation grade, when approaching evaluation criterion, *a*, b_1 more and more big, c_1 more and more small; if index value is in separated grade, and on inferior side of evaluation grade, when approaching evaluation criterion, *a*, b_2 more and more big, c_2 more and more small.

Setting up five grades index evaluation model of more less more better is given in Box 1.

In the model, $v_{1k}, v_{2k}, v_{3k}, v_{4k}, v_{5k}$ are the limit value of grade I, gradea II, grade III, grade IV, grade V respectively, x_{qk} is No. k index value of the q region groundwater environment. According to formula (4) calculate the value of relation degree coefficient a, b_1, b_2, c_1, c_2 of every index on each region, and then evaluation by combining each index weight value.

EXAMPLE OF CALCULATION

According to researching region state, we have selected four evaluation factors which affect water environment as follows: COD_{Mn}, DO, NH_3-N, F^-. Adopting test result of seven water samples (X_1: Lengkou; X_2: Yakou; X_3: Qiuzhuang reservoir; X_4: Xiaodingfu city; X_5: Douhe reservoir; X_6: Shifekou; X_7: Luanxian city) to calculate and evaluate result of water quality analysis (Table 1).

According to “Surface Water Environment Quality Criterion” (GB 3838-2002) to divide grade, grade criterion see Table 2.

Using entropy method to ensure every index weight (Wei 2004, Wei 2009):

1. According to fact measure value of Table 1 to set up original data matrix $X = (x_{ij})_{7\times 4}$:

Box-1

Relation degree relative to the first grade:

$$u_1\left(v_{1k},x_{qk}\right)=a+b_1i^+ +b_2i^- +c_1j^+ +c_2j^- =\begin{cases} 1 & [0,v_{1k}) \\ \dfrac{v_{1k}}{x_{qk}}+\dfrac{x_{qk}-v_{1k}}{x_{qk}}i^- & [v_{1k},v_{2k}) \\ \dfrac{v_{1k}}{x_{qk}}+\dfrac{v_{2k}-v_{1k}}{x_{qk}}i^- +\dfrac{x_{qk}-v_{2k}}{x_{qk}}j^- & [v_{2k},+\infty) \end{cases} \quad (4)$$

Relation degree relative to the second grade:

$$u_2\left(v_{2k},x_{qk}\right)=a+b_1i^+ +b_2i^- +c_1j^+ +c_2j^- =\begin{cases} \dfrac{v_{2k}-v_{1k}}{v_{2k}-x_{qk}}+\dfrac{v_{1k}-x_{qk}}{v_{2k}-x_{qk}}i^+ & [0,v_{1k}) \\ 1 & [v_{1k},v_{2k}) \\ \dfrac{v_{2k}-v_{1k}}{x_{qk}-v_{1k}}+\dfrac{x_{qk}-v_{2k}}{x_{qk}-v_{1k}}i^- & [v_{2k},v_{3k}) \\ \dfrac{v_{2k}-v_{1k}}{x_{qk}-v_{1k}}+\dfrac{v_{3k}-v_{2k}}{x_{qk}-v_{1k}}i^- +\dfrac{x_{qk}-v_{3k}}{x_{qk}-v_{1k}}j^- & [v_{3k},+\infty) \end{cases}$$

Relation degree relative to the third grade:

$$u_3\left(v_{3k},x_{qk}\right)=a+b_1i^+ +b_2i^- +c_1j^+ +c_2j^- =\begin{cases} \dfrac{v_{3k}-v_{2k}}{v_{3k}-x_{qk}}+\dfrac{v_{2k}-v_{1k}}{v_{3k}-x_{qk}}i^+ +\dfrac{v_{1k}-x_{qk}}{v_{3k}-x_{qk}}j^+ & [0,v_{1k}) \\ \dfrac{v_{3k}-v_{2k}}{v_{3k}-x_{qk}}+\dfrac{v_{2k}-x_{qk}}{v_{3k}-x_{qk}}i^+ & [v_{1k},v_{2k}) \\ 1 & [v_{2k},v_{3k}) \\ \dfrac{v_{3k}-v_{2k}}{x_{qk}-v_{2k}}+\dfrac{x_{qk}-v_{3k}}{x_{qk}-v_{2k}}i^- & [v_{3k},v_{4k}) \\ \dfrac{v_{3k}-v_{2k}}{x_{qk}-v_{2k}}+\dfrac{v_{4k}-v_{3k}}{x_{qk}-v_{2k}}i^- +\dfrac{x_{qk}-v_{4k}}{x_{qk}-v_{2k}}j^- & [v_{4k},+\infty) \end{cases}$$

Box 1 Cont....

Cont.... Box 1

Box-1

Relation degree relative to the fourth grade:

$$u_4\left(v_{4k}, x_{qk}\right) = a + b_1 i^+ + b_2 i^- + c_1 j^+ + c_2 j^- = \begin{cases} \dfrac{v_{4k} - v_{3k}}{v_{4k} - x_{qk}} + \dfrac{v_{3k} - v_{2k}}{v_{4k} - x_{qk}} i^+ + \dfrac{v_{2k} - x_{qk}}{v_{4k} - x_{qk}} j^+ & [0, v_{2k}) \\ \dfrac{v_{4k} - v_{3k}}{v_{4k} - x_{qk}} + \dfrac{v_{3k} - x_{qk}}{v_{4k} - x_{qk}} i^+ & [v_{2k}, v_{3k}) \\ 1 & [v_{3k}, v_{4k}) \\ \dfrac{v_{4k} - v_{3k}}{x_{qk} - v_{3k}} + \dfrac{x_{qk} - v_{4k}}{x_{qk} - v_{3k}} i^- & [v_{4k}, +\infty) \end{cases}$$

Relation degree relative to the fifth grade:

$$u_5\left(v_{5k}, x_{qk}\right) = a + b_1 i^+ + b_2 i^- + c_1 j^+ + c_2 j^- = \begin{cases} \dfrac{v_{5k} - v_{4k}}{v_{5k} - x_{qk}} + \dfrac{v_{4k} - v_{3k}}{v_{5k} - x_{qk}} i^+ + \dfrac{v_{3k} - x_{qk}}{v_{5k} - x_{qk}} j^+ & [0, v_{3k}) \\ \dfrac{v_{5k} - v_{4k}}{v_{5k} - x_{qk}} + \dfrac{v_{4k} - x_{qk}}{v_{5k} - x_{qk}} i^+ & [v_{3k}, v_{4k}) \\ 1 & [v_{4k}, +\infty) \end{cases}$$

$$X = (x_{ij})_{7\times 4} = \begin{bmatrix} 1.45 & 9.13 & 0.23 & 0.30 \\ 2.62 & 8.27 & 0.33 & 0.60 \\ 5.73 & 8.93 & 0.26 & 0.48 \\ 34.12 & 0.70 & 16.98 & 0.41 \\ 3.66 & 8.63 & 0.13 & 0.63 \\ 2.60 & 10.00 & 0.96 & 0.58 \\ 38.03 & 3.83 & 2.72 & 0.41 \end{bmatrix}$$

2. Dealing with data x_{ij}. According to the formula (Wu 2001):

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad i = 1, 2, \mathbf{L}, m;\ j = 1, 2, \mathbf{L}, n$$

So,

$$X' = (p_{ij})_{7\times 4} = \begin{bmatrix} 0.016 & 0.184 & 0.011 & 0.088 \\ 0.230 & 0.167 & 0.015 & 0.176 \\ 0.065 & 0.180 & 0.012 & 0.141 \\ 0.387 & 0.014 & 0.786 & 0.120 \\ 0.042 & 0.174 & 0.006 & 0.185 \\ 0.029 & 0.202 & 0.044 & 0.170 \\ 0.431 & 0.077 & 0.126 & 0.120 \end{bmatrix}$$

Table1: Result of water quality analysis. unit:mg/L

Spots	COD_{Mn}	DO	NH_3-N	F^-
X_1	1.45	9.13	0.23	0.30
X_2	2.62	8.27	0.33	0.60
X_3	5.73	8.93	0.26	0.48
X_4	34.12	0.70	16.98	0.41
X_5	3.66	8.63	0.13	0.63
X_6	2.60	10.00	0.96	0.58
X_7	38.03	3.83	2.72	0.41

3. Calculating No. j index information entropy:

$$e_j = -\sum_{i=1}^{m} p_{ij} \ln p_{ij} \qquad i = 1, 2, \mathbf{L}, m;\ j = 1, 2, \mathbf{L}, n$$

According to the above formula:

$e_1 = 1.806; e_2 = 0.785; e_3 = 1.316; e_4 = 1.918$

4. Calculating No. j index weight. For index j, its weight more big, evaluation index discrepancy more big, nevertheless, evaluation index entropy more little, so order:

$$u_j = \frac{1}{e_j} \qquad j = 1, 2, \mathbf{L}, n$$

Table 2: Surface water environment quality criterion grade. unit:mg/L.

Dividing grade	COD_{Mn}(≤)	DO (≥)	NH_3-N(≤)	F^-(≤)
I	2	7.5	0.15	1.0
II	4	6	0.5	1.0
III	6	5	1.0	1.0
IV	10	3	1.5	1.5
V	15	2	2.0	1.5

According to the above formula:

$$u_1 = 0.554; u_2 = 1.274; u_3 = 0.760; u_4 = 0.521$$

Dealing with data u_j, so as to $\sum w_j = 1$ .Therefore, No. j index weight is $w_j = \frac{u_j}{\sum_{j=1}^{n} u_j}$, namely:

$$w_1 = 0.178; w_2 = 0.410; w_3 = 0.244; w_4 = 0.168$$

So, we can draw weight vector:

$$W = [w_1 \quad w_2 \quad w_3 \quad w_4] = [0.178 \quad 0.410 \quad 0.244 \quad 0.168]$$

Improved SPA fuzzy comprehensive evaluation calculation

1. Setting up and calculation of *SPA* relation degree matrix of R_1 of spot X_1

$$R_1 = \begin{bmatrix} 1 & 0.784+0.216i^{+} & 0.440+0.440i^{+}+0.120j^{+} & 0.468+0.234i^{+}+0.298j^{+} & 0.369+0.295i^{+}+0.336j^{+} \\ 1 & 0.479+0.521i^{+} & 0.242+0.363i^{+}+0.395j^{+} & 0.326+0.163i^{+}+0.511j^{+} & 0.140+0.282i^{+}+0.578j^{+} \\ 0.652+0.348i^{-} & 1 & 0.649+0.351i^{+} & 0.394+0.394i^{+}+0.212j^{+} & 0.283+0.283i^{+}+0.434j^{+} \\ 1 & 1i^{+} & 1j^{+} & 0.417+0.583j^{+} & 0.417i^{+}+0.583j^{+} \end{bmatrix}$$

2. Carrying out compounding operation $b_1 = W \circ R_1$:

Applying Matlab calculation software for operation on spot X_1, as:

$$B_1 = W \circ R_1 = [0.9151 + 0.0849i^{-} \quad 0.5801 + 0.4199i^{+} \quad 0.336 + 0.3127i^{+} + 0.3513j^{+} \quad 0.3831 + 0.2046i^{+} + 0.4123j^{+} \quad 0.1921 + 0.307i^{+} + 0.5009j^{+}]$$

Therefore, $shi(A) = a/c$ or $shi(A) = a/b$ (Zhao 2006):

$shi(u_1) = 10.779$ $shi(u_2) = 1.382$ $shi(u_3) = 0.956$
$shi(u_4) = 0.929$ $shi(u_5) = 0.384$

Five grade set pair situation normalization operation, the probability distribution of measure spot X_1 from grade I to grade V, respectively are: 74.7%, 09.6%, 06.6%, 06.4% and 02.7%. So, evaluation result on spot X_1 is grade I . The same, we can draw evaluation result of others six measure spots, from Tables 3 and 4.

Comparison of composite index method and gray correlation analysis method (Zhao 1996) is given in in Table 4.

Table 3: Spots evaluation result.

Spot-s	*shi* (u_1)	*shi* (u_2)	*shi* (u_3)	*shi* (u_4)	*shi* (u_5)
X_1	10.779	1.382	0.956	0.929	0.384
X_2	4.708	2.256	1.414	1.405	0.483
X_3	14.515	1.219	1.410	1.448	0.528
X_4	0.289	0.034	0.033	1.670	9.153
X_5	11.392	1.798	1.346	1.311	0.461
X_6	6.442	0.777	1.155	1.980	0.614
X_7	0.790	0.290	0.468	6.383	7.789

Table 4: Evaluation result contrast.

Spots	Composite index method	Gray correlation analysis	Improved fuzzy comprehensive evaluation
X_1	I	I	I
X_2	I	I	I
X_3	I	I	I
X_4	V	V	V
X_5	I	I	I
X_6	I	I	I
X_7	V	IV	V

CONCLUSIONS

1. Using entropy method to ensure weight is objective, it avoids artificial interference as expert's marks etc. subjective giving to weight method.
2. Improved method is more detailed on identity, discrepancy and contrary character of matter, and avoiding discussion about coefficient value, at the same time, we utilize original relation degree malleability of to set up five grade evaluation new model of surface water environment quality.
3. Improved method combines with fuzzy comprehensive evaluation method and apply on calculation example of surface water environment evaluation on a certain region Hebei Province, drawing the better effect, and comparing with composite index method and gray correlation analysis method, the evaluation result is the most important part or meaning accordant. In addition, this method may get probability distribution state in every evaluation grade, it provide a kind of more scientific, rational evaluation and decision method for surface water environment evaluation, having the better application value.

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